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Early Experiments with Traffic System Model and Factorized Kalman Filtering

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Abstract *- The paper deals with the factorized version of the Kalman filtering. The global aim of research is to make possible to estimate the entries of the state vector individually and thereby to approach to handling the mixed-typed states, which still remains the open problem. The present paper continues a sequence of research in the field of factorized filtering and demonstrates its use with the urban traffic system model, which is the main application area. Two illustrative examples are presented.*

Keywords *- State-space model, State estimation, Urban traffic control, Factorized filtering*

I. Introduction

The paper deals with the factorized version of Kalman filtering [1] and describes its application to traffic control. The traffic situation in the cities, along with a permanently growing number of cars and, at the same time, the queues at the intersections, has been a motivation for the presented work. The people have to waste their time at queues on the intersections, not to mention ecology. The situation is much worse in historical cities, where extension of traffic network is expensive and often impossible. One of the steps towards improving the situation might be the use of means of modern feedback control with a queue length, taken as the main controlled variable. The length of the queue expresses a state of the transport network most adequately, but it is not directly observable and has to be estimated. The estimation of the individual state vector entries (*factors*), which the factorized version of filtering brings, is believed to enable approaching to the global aim of the research – handling the mixed-type states, for the time remaining the open problem.

The present paper continues a sequence of the research works, devoted to the factorized state estimation. The paper [2] was concerned with the entry-wise organized filtering under Bayesian methodology [3] and proposed the recursive algorithm. But it required a special, reduced, form of the state-space model, which caused excessive restrictions. The work [4] proposed the solution of factorized Bayesian prediction and filtering, based on applying the chain rule to the state-space model, without such a restriction. Bayesian filtering with Gaussian state-space model, Gaussian prior distribution of the initial state and Gaussian observations provides Kalman filter. The work [5] described the problem of the factorized Kalman filtering with Gaussian models and offered the solution, based on applying the $L'DL$ decomposition of the covariance matrix. The present papers expands this line of research by demonstrating the application of the solution to the traffic system state-space model.

The layout of the paper is organized in the following way. Section II provides the necessary basic facts from urban traffic control and describes the model of the traffic situation at the arm (or arms) of the intersection. Section III is devoted to the factorized version of Kalman filtering, applied to the normal traffic system model. Section IV demonstrates the experiments with the factorized Kalman filter and the traffic model. The experiments use the data from realistic traffic simulation for the case with one and two intersections with four arms. The remarks in Section V close the paper.

II. Traffic system model

A. Basic Facts of Traffic Control

In order to describe an intersection with one or several arms by a model, it is necessary to introduce some basic concepts of traffic control [6]. The controlled intersection is supposed to be equipped by the measuring *detectors*. Their number and location can vary because of the road conditions, but at this paper it is assumed, that the *input detectors* are placed about 100 m before the stop line at the input lane of the intersection arm, and the output lanes are equipped by the *output detectors*.

The detectors measure the quantities, which are of importance for the intersection modelling. The quantities are as follows.

- *Intensity*, expressing a number of the passing cars per hour $[c/h]$.
- *Occupancy*, reflecting a proportion of a time period of activating the detector by cars $[\%]$.

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The observed quantities are influenced by such the control variables as *cycle time* of the traffic signal, meaning a period of a phase exchange, and time of a green light in one direction in seconds $[s]$. At the paper these variables are taken as the known inputs. The intersection lane is also characterized by such a quantity as the *saturated flow*, which expresses the maximal number of cars, which can pass through the lane per hour in the case of the green light $[c/h]$.

The queue length ξ_t at the intersection arm is the main state variable at discrete time moments $t \in t^* \equiv \{1, \ldots, \mathring{t}\}$, that has to be estimated. Throughout the paper the queue length is considered in meters $[m]$. The general idea of its evolution lies in the statement, that the queue length is equal to the previous queue plus arrived cars minus departed cars. It can be expressed in the following way.

$$\xi_{t+1} = \delta_t \xi_t - \underbrace{[\delta_t S + (1-\delta_t) I_t] z_t}_{departed\ cars} + \underbrace{I_t}_{arrived}, \quad (1)$$

where S is the saturated flow; I_t is the input intensity; z_t is a green time; δ_t is a time-varying parameter so that $\delta_t = 1$ if the queue exists and $\delta_t = 0$ otherwise. The parameter δ_t should be described in more detail. The presence of δ_t in the traffic model reflects two following important situations, that can occur at the intersection lane.

- The first one is the queue length and the input intensity are relatively small. The output intensity of the intersection lane includes the arrived cars (input intensity) and the previous queueing cars. In such a case all the cars can pass through the lane (lanes) without queue and $\delta_t = 0$. This relation can be intuitively understood from (1).
- The second case happens, when the input intensity along with the previous queue length is too big. It leads to the situation, that only the saturated flow of the cars during the green time can pass through the intersection lane, and the queue occurs ($\delta_t = 1$).

B. Model of the Intersection Arm

The state-space model, used at the paper, has the following form.

$$x_{t+1} = A_t x_t + B_t z_t + F_t + w_t, \quad (2)$$
$$y_{t+1} = C x_{t+1} + G_t + v_{t+1}, \quad (3)$$

where x_t is the system state, y_t – the system output, A_t, B_t, C are matrices with parameters, F_t and G_t are specific traffic vectors, defined below. The process noise w_t and the measurement noise v_t are Gaussian ones and have zero means and known covariances R_w and R_v respectively. In order to describe the traffic situation at *one arm* of the intersection, the model (2)-(3) can be treated as [7]

$$\underbrace{\begin{bmatrix} \xi_{t+1} \\ O_{t+1} \end{bmatrix}}_{x_{t+1}} = \underbrace{\begin{bmatrix} \delta_t & 0 \\ \kappa & \beta \end{bmatrix}}_{A_t} \cdot \underbrace{\begin{bmatrix} \xi_t \\ O_t \end{bmatrix}}_{x_t} + \underbrace{\begin{bmatrix} -\delta_t \cdot S - (1-\delta_t) \cdot I_t \\ 0 \end{bmatrix}}_{B_t} \cdot z_t + \underbrace{\begin{bmatrix} I_t \\ \lambda \end{bmatrix}}_{F_t} + w_t, \quad (4)$$

$$\underbrace{\begin{bmatrix} Y_{t+1} \\ O_{t+1} \end{bmatrix}}_{y_{t+1}} = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \cdot \underbrace{\begin{bmatrix} \xi_{t+1} \\ O_{t+1} \end{bmatrix}}_{x_{t+1}} + \underbrace{\begin{bmatrix} \xi_t + I_t \\ 0 \end{bmatrix}}_{G_t} + v_{t+1}, \quad (5)$$

where the state x_t contains not only the unobserved queue length, but also the measurable occupancy O_t, which is added to ensure the observability of the model. The utilization of the occupancy in the model expresses its proportionality to the length of the queue. The parameters κ, β, λ are generally time-variant. They reflect a character of traffic during a week and weekend as well as the main peak hours during a day. The measurable output y_t is given by $y_t = [Y_t, O_t]'$, where Y_t is the output intensity, i. e. the intensity measured by the output detector, $'$ is a transposition sign.

C. Intersection Model

The model of the intersection is modified according to the number of the arms n, that is reached by adding the index $i = 1, \ldots, n$ to (4)-(5). Thus, the state equation (4) of the model of the intersection with n arms takes the following form.

$$\begin{bmatrix} \xi_{1,t+1} \\ \vdots \\ \xi_{n,t+1} \\ O_{1,t+1} \\ \vdots \\ O_{n,t+1} \end{bmatrix} = \begin{bmatrix} \Delta_t & \mathcal{Z} \\ \mathcal{K}_t & \mathcal{B}_t \end{bmatrix} \cdot x_t + \begin{bmatrix} -\tilde{B}_t \\ \mathcal{Z} \end{bmatrix} \cdot z_t + \begin{bmatrix} I_{1,t} \\ \vdots \\ I_{n,t} \\ \lambda_{1,t} \\ \vdots \\ \lambda_{n,t} \end{bmatrix} + w_t, \quad (6)$$

where Δ_t, $\mathcal{K}_t$, $\mathcal{B}_t$ and $\tilde{B}_t$ are the diagonal matrices of appropriate dimensions with elements $\delta_{i,t}$, $\kappa_{i,t}$, $\beta_{i,t}$ and $b_{i,t} = \delta_{i,t}S_i + (1-\delta_{i,t})I_{i,t}$ respectively at the main diagonal. For example, matrix Δ_t has a form

$$\Delta_t = \begin{bmatrix} \delta_{1,t} & 0 & \cdots & 0 \\ 0 & \delta_{2,t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \delta_{n,t} \end{bmatrix}. \quad (7)$$

The rest of the mentioned matrices have an analogous form with the corresponding elements. Matrix $\mathcal{Z}$ is a zero square matrix of dimension $(n \times n)$.

The output equation (5) requires to be slightly modified. Modelling the intersection, one should introduce the additional parameter *turn rate* $\alpha_{i,j}$, which describes a ratio of the cars, going from the i-th arm and turning to the j-th arm. This parameter is assumed to be known and constant and can be usually obtained from the traffic expert information of the considered region. Thus, the output equation for the intersection is

$$\begin{bmatrix} Y_{1,t+1} \\ \vdots \\ Y_{n,t+1} \\ O_{1,t+1} \\ \vdots \\ O_{n,t+1} \end{bmatrix} = \begin{bmatrix} -\mathcal{M} & \mathcal{Z} \\ \mathcal{Z} & 1 \end{bmatrix} \cdot x_{t+1} + \begin{bmatrix} \mathcal{M} & \mathcal{Z} \\ \mathcal{Z} & 1 \end{bmatrix} \times \begin{bmatrix} \xi_{1,t} + I_{1,t} \\ \vdots \\ \xi_{n,t} + I_{n,t} \\ 0 \end{bmatrix} + v_{t+1}, \quad (8)$$

where 1 is a unit matrix of the appropriate dimension, 0 is a zero column vector and $\mathcal{M}$ includes the parameters of the turn rate in the following way.

$$\mathcal{M} = \begin{bmatrix} 0 & \alpha_{21} & \alpha_{31} & \cdots & \alpha_{n1} \\ \alpha_{12} & 0 & \alpha_{32} & \cdots & \alpha_{n2} \\ \alpha_{13} & \alpha_{23} & 0 & \cdots & \alpha_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_{1n} & \alpha_{2n} & \alpha_{3n} & \cdots & 0 \end{bmatrix} \quad (9)$$

It should be noted, that the output equation (5), or (8), reflects the point, that the output intensity is equal to the number of cars passed through the intersection arm. It explains the negative queue length at time moment $t+1$ and positive at moment t in (8). In this way, the model of the intersection with n arms includes equations (6)-(8).

III. Factorized Kalman filtering

The factorized version of Kalman filtering, proposed in [5], assumes the use of the $L'DL$-decomposed covariance matrices, where L is lower triangular matrix with unit diagonal and D is a diagonal matrix. When applying the factorized filter to the traffic state-space model (2)-(3), one can present the model in the following way.

$$x_{t+1} = A_t x_t + B_t z_t + F_t + W'\omega_t, \quad (10)$$
$$y_{t+1} = C x_{t+1} + G_t + V' v_{t+1}, \quad (11)$$

where $W'R_wW$ and $V'R_vV$ are $L'DL$ decomposed matrices. The state estimate is assumed to be calculated with $L'DL$-factorized covariance matrix, i. e. as $\mathcal{N}(\hat{x}_{t+1}; P_{t+1})$, where $P_{t+1} = L'_{P(t+1)} D_{P(t+1)} L_{P(t+1)}$. The initial values of mean $\hat{x}_0$ and covariance $P_0 = L'_0 D_0 L_0$ are known.

The algorithm of the factorized Kalman filter includes the coupled procedures of time updating and data updating as follows (for acquaintance with the *Kalman* filter a reader is referred to [1]).

Time updating

$$W'^{-1}\hat{x}^-_{t+1} = W'^{-1}A_t\hat{x}_t + W'^{-1}B_t z_t + W'^{-1}F_t, \quad (12)$$
$$P^-_{t+1} = A_t P_t A'_t + W'R_wW, \quad (13)$$

Data updating

$$K_{t+1} = P^-_{t+1}C'(CP^-_{t+1}C' + V'R_vV)^{-1}, \quad (14)$$
$$P_{t+1} = ({P^-_{t+1}}^{-1} + C'(V'R_vV)^{-1}C)^{-1}, \quad (15)$$
$$\hat{x}_{t+1} = \hat{x}^-_{t+1} + K_{t+1}(y_{t+1} - C\hat{x}^-_{t+1} - G_t). \quad (16)$$

Several explaining remarks should be made here. Time updating step predicts the state mean $\hat{x}^-_{t+1}$ and covariance matrix P^-_{t+1} (the superscript "-" denotes a priori estimate). Multiplication of (12) by inverse matrix W'^{-1} enables holding a triangular structure of matrices. With the help of introducing the new state vector $\tilde{\hat{x}}_t = A_t\hat{x}_t$ and the new vector of the green time $\tilde{z}_t = B_t z_t$, the triangular structure becomes obvious and can be used for individual approach to the state entries. The structure can be schematically demonstrated as follows.

$$\underbrace{\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot \\ & & \cdot & \cdot \\ & & & \cdot \end{bmatrix}}_{W'^{-1}} \underbrace{\begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}}_{\hat{x}^-_{t+1}} = \underbrace{\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot \\ & & \cdot & \cdot \\ & & & \cdot \end{bmatrix}}_{W'^{-1}} \underbrace{\begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}}_{\tilde{\hat{x}}_t} + W'^{-1}\tilde{z}_t + W'^{-1}F_t.$$

It can be shown, that denoting $W'^{-1} = H$ for shorter notion, one can obtain the following relation for the predicted estimate of the i-th state factor with dimension of the state vector, equal to $2n$. H is the upper triangular matrix with unit diagonal.

$$\hat{x}^-_{i(t+1)} + \sum_{k=i+1}^{2n} h_{ik}\hat{x}^-_{k(t+1)} \quad (17)$$

$$= \tilde{\hat{x}}_{i(t)} + \sum_{k=i+1}^{2n} h_{ik}\tilde{\hat{x}}_{k(t)} + \tilde{z}_{i(t)} + \sum_{k=i+1}^{2n} h_{ik}\tilde{z}_{k(t)}$$

$$+ \quad f_{i(t)} + \sum_{k=i+1}^{2n} h_{ik} f_{k(t)},$$

$$\hat{x}^-_{i(t+1)} = \tilde{\hat{x}}_{i(t)} + \sum_{k=i+1}^{2n} h_{ik}(\tilde{\hat{x}}_{k(t)} - \hat{x}^-_{k(t+1)})$$

$$+ \quad \tilde{z}_{i(t)} + f_{i(t)} + \sum_{k=i+1}^{2n} h_{ik}(\tilde{z}_{k(t)} + f_{k(t)}). \quad (18)$$

The updating of the covariance matrix in (13) is performed as $L'^-_{P(t+1)} D^-_{P(t+1)} L^-_{P(t+1)}$ without forming this product. The calculation exploits the algorithms from the toolbox Mixtools [8] and does not contain numerically dangerous operations.

As regards the data updating procedure, it corrects the state estimate by incorporating the information, contained in the input and output intensities. The calculation of the covariance matrix is based on the matrix inversion lemma [9]. Exploitation of the mentioned lemma in (15) allows obtaining the elegant solution for the efficient update of covariance matrix P_{t+1} as $L'_{P(t+1)} D_{P(t+1)} L_{P(t+1)}$. The Kalman gain K_{t+1} is used in calculation of the posterior mean $\hat{x}_{t+1}$ in (16). The mean of the individual i-th state entry takes a form

$$\hat{x}_{i(t+1)} \quad = \quad \hat{x}^-_{i(t+1)} + \sum_{j=1}^{2n} k_{ij(t+1)}\gamma_{j(t+1)}, \quad (19)$$

where the known column vector

$$\gamma_{t+1} = y_{t+1} - C\hat{x}^-_{t+1} - G_t. \quad (20)$$

The schematic representation of (16) is shown below.

$$\underbrace{\begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}}_{\hat{x}_{t+1}} = \underbrace{\begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}}_{\hat{x}^-_{t+1}} + \underbrace{\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}}_{K_{t+1}} \underbrace{\begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}}_{\gamma_{t+1}}.$$

IV. ILLUSTRATIVE EXPERIMENTS

A. Experiment 1 – Intersection

The factorized version of the state estimation has been tested on the intersection with four arms. All the input and output lanes of the intersection are supposed to be equipped by the detectors and, therefore, are measured. A scheme of the intersection is shown at Fig. 1. Fig. 1 demonstrates, that

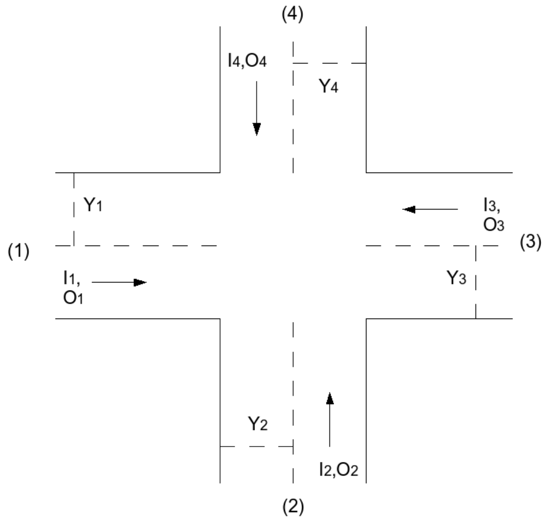

Figure 1: The intersection with four measured arms

every input lane provides the observed input intensity $I_{i,t}$ and occupancy $O_{i,t}$, while the i-th output lane – the output intensity $Y_{i,t}$ at time moment t respectively.

The modelling of the intersection has been based on the data, simulated by the traffic microsimulator AIMSUN [10]. The software AIMSUN is able to simulate very realistic traffic situations, taking into account the region conditions practically of any traffic network. 4800 data, simulated by AIMSUN, have been taken for the experiment. The time-varying parameters $\kappa_{i,t}$, $\beta_{i,t}$, $\lambda_{i,t}$, forming the matrices of parameters in the model 2)-(3), have been estimated off-line for some period of time and then were used for running the filter. The parameter $\delta_{i,t}$ has been set so that it would indicate whether the queue is forming according to the simulated input intensities and saturated flows.

The filter has been run and estimated the queue lengths. The estimated lengths of the queues have been compared with the simulated ones. The result can be seen at Fig. 2. The order of the arm positions at Fig. 2 is the same as the arms appear at Fig. 1. The queue on the i-th, $i = \{1, \ldots, n\}$, arm of the intersection is concerned to the i-th entry of the unobservable part of the state vector $[\xi_{1,t} \ldots \xi_{n,t}]'$. Fig. 2 demonstrates the good correspondence of the simulated and estimated values.

The estimated values have been compared with the results, obtained from the Kalman filter. The obtained state mean values and the covariance matrices (after multiplying the $L'DL$-factorized matrices) showed very insignificant difference and confirmed the functioning the algorithm with the traffic model. The result of the whiteness test [11]

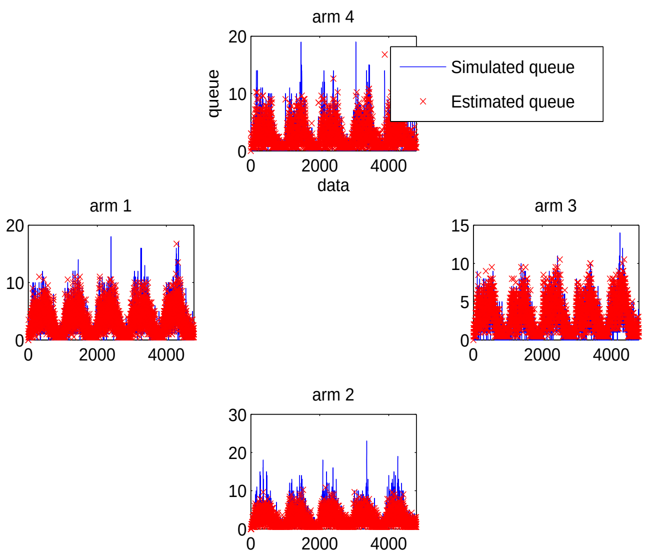

Figure 2: Factorized Kalman filter for the intersection

of the prediction error is 0.9883, which demonstrates, that there is no more information to be extracted from the data.

B. Experiment 2 – Microregion

The second experiment of application of the factorized Kalman filter has been held with a *microregion*. The considered simulated microregion includes two interconnected intersections, each one with four arms. In practice it is very usual situation, when the microregion has some unmeasured input and output lanes. At the experiment two internal arms at the point of connection of the intersections are not equipped by detectors and not measurable. The rest of the arms are supposed to be measured by detectors. The

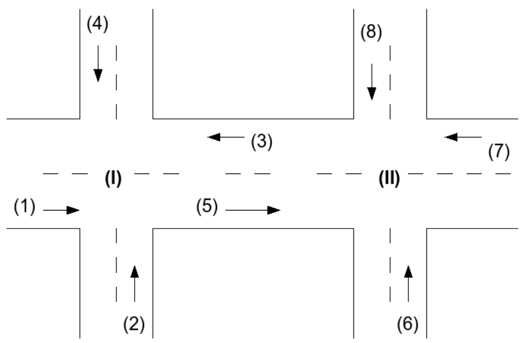

Figure 3: The microregion

scheme of the microregion is plotted at Fig. 3, where the Roman numerals indicate the intersections and the Arabian numerals correspond to the numbers of the input lanes. The lanes (3) and (5) are placed between the intersections and not measured. About 300 data have been simulated for the experiment. At the lanes, where the intensities and occupancies were assumed not to be observed, the zero arrays have been used instead of the simulated quantities.

The results of the queue length estimation at the arms 1 and 6 respectively and comparison with the simulated queues are shown at Fig. 4. The estimation for all the arms, which have been measured, gave the similar adequate results.

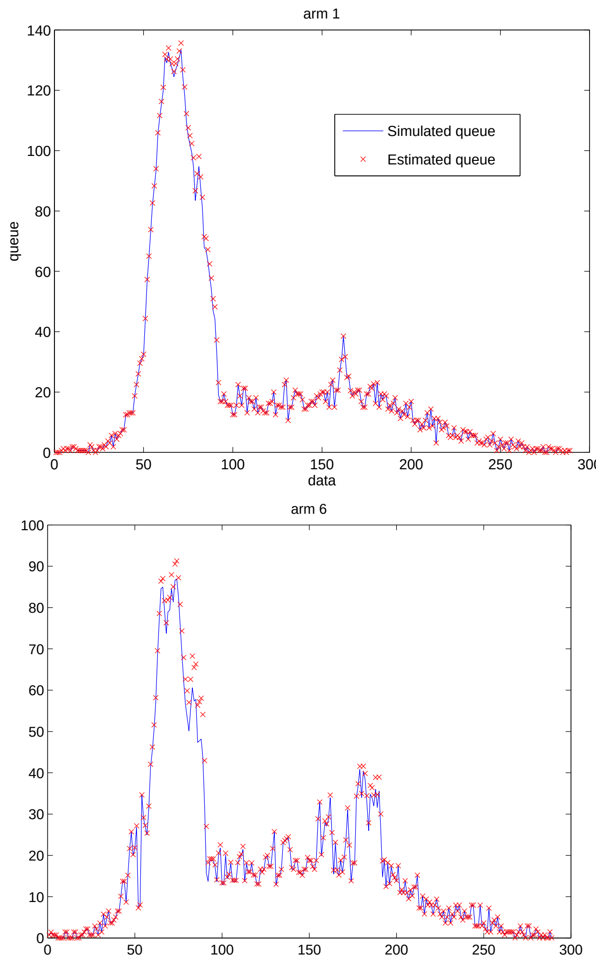

Figure 4: Simulated and estimated queues at arms 1 and 6

Fig. 5 demonstrates the results of the queue estimation at the unmeasured intersection lanes (3) and (5). It can be seen, that the correspondence between the simulated and the estimated values is a little bit worse, than for the measured input lanes. The estimation problem of the microregion with the unmeasured lanes is usually solved in two following ways. The unmeasured intensities and occupancies can be either added to the process noise, if they are

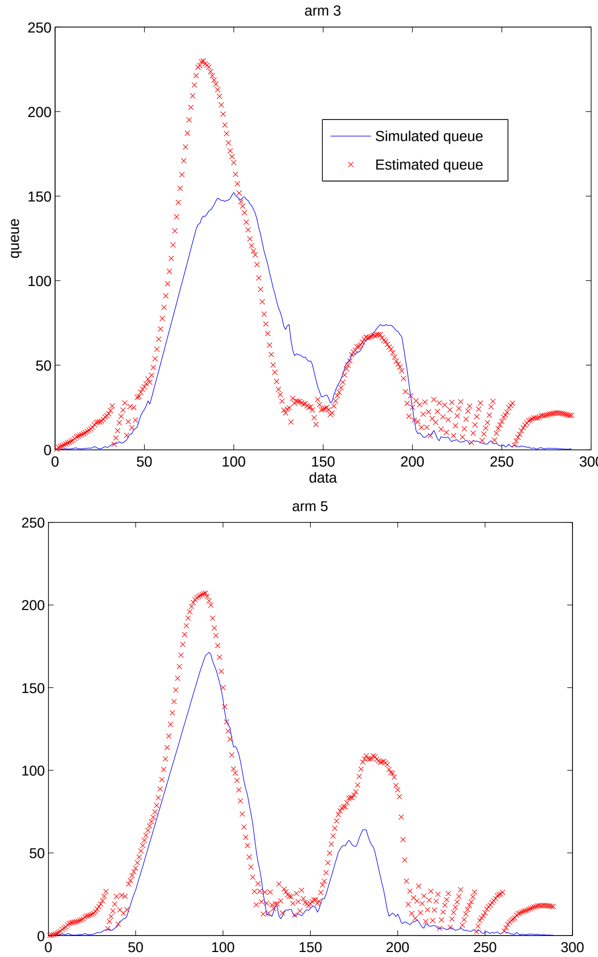

Figure 5: Estimation results at the unmeasured arms

not significant for the whole microregion, or they should be estimated. In the second case the model requires the modifying the state equation by involving the unmeasured quantity, which usually causes the extension of the state vector and increasing its dimension. At the presented experiment the presence of the detector, for example, at the lane (5) is not necessary, because the input intensities at the lanes (1), (2) and (4) are measured and the parameter of the turn rate is known. The similar situation is with the lane (3). In this way, the unmeasured quantities here can be considered as the noise, including the inaccuracy of the model and observations. However, the unmeasured lane, for example, (2), would significantly influence the output intensity on the lane (1) as well as input and output intensities on the lane (5). In this case it would be necessary to estimate the unmeasured intensity.

The whiteness test of the prediction error at the present experiment results in value 0.9987.

V. Conclusion

The paper is devoted to the application of the factorized version of the Kalman filter [5] to the traffic state-space model. The factorized filter is directed at the individual estimation of the entries (factors) of the state vector and is expected to help reaching the global aim of the research – mixed-typed states estimation, still remaining the open problem. The present paper continues a sequence of research in this area and demonstrates the estimation of the queue length, used as the main state variable in the model of the urban traffic system. The description of the traffic system model is presented for the case with the intersection arm and for the intersection with n arms. The application of the filter is demonstrated as the theoretical solution as well as by means of illustrative examples with the simulated intersections.

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