State Space Model of a Traffic Microregion

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Abstract. The paper deals with estimation of queue lengths and unknown parameters of state space model of a microregion. Crossroads in cities are controlled by setting a proper green proportion and cycle length of the signal lights. Mostly, these signals are set according to a working plan whose changes during the day are given by a fixed schedule.

Automatic feedback in selection of these plans can improve quality of the traffic significantly. For the control, knowledge of traffic flow state incoming into the microregion in the near future is necessary. Reliable and possibly multi-step prediction of the traffic flow can decide about practical success of such a feedback control.

Keywords
Traffic flow control, basic transportation quantities, queue length model, estimation, Kalman filter.

1. Introduction

Majority of cities has problem with high density of transportation and a large amount of junctions and roads are congested. In many cities these structures cannot easily accommodate the vast volume of traffic, which results in regular traffic congestions. The most common approaches to solution of these transport network deficiencies are attempts to rebuild parts of the network in order to increase their capacity, reroute transit traffic by building large bypasses around affected locations or making drivers pay for entering central zones.

In many cities it is impossible or ineffective to reconstruct the existing street network due to their historical development. This is the reason why the advanced traffic control is being applied.

The suggested model describes the queue length in a linear way and it is supposed to be used for traffic control which minimizes the weighted sum of queue lengths on all junction arms. The proposed model counts and estimates the queue length on the basis of maximum given traffic information. This task is trivia in case of complete knowledge of all measured traffic quantities for all junction arms. Then the model simply counts the queue length from known input and output intensities. However, the net of all needed detectors is not usually complete and some significant traffic flows (parking cars, etc.) are not measurable in practice. In this case, the model estimates the queue length relative to modelled and estimated traffic characteristics.

2. Notations, terms and quantities

2.1 Traffic data

Controlled networks are split into microregions. They are logically self-contained transportation areas of several crossroads with their adjoining roads. Their modeling and feedback control exploit data measured by detectors based on inductive electric coils placed under the road surface. Presence of a huge metallic object above the coil changes its magnetic properties and thus individual cars are detected. Each detector signalizes presence or absence of a car above it. From this signal, basic transportation quantities are evaluated:

Occupancy: determines the relative time of the detector activation during the sample period, i.e. the proportion of time when the detector has been occupied and the total time of measuring period. The occupancy unit is [%]. This quantity has similar meaning as the density. The higher density decreases the vehicles velocity and intensities and queues are formed on the arms. That is why the value of occupancy of the detectors under the queue increases. Conversely, if the vehicles can go through faster in case of low traffic and they loose the minimum time by queueing, the occupancy decreases.

Intensity: denotes the number of vehicles which have passed a detector during the sample period. Usually, the value of this quantity is transformed into an hourly intensity of unit vehicles, i.e. [uv/h]. This quantity captures the queue dynamics in a sense of the queue protraction but it does not fully determine the actual situation. The value
of intensity can be low because of low traffic or high density which are two converse traffic situations. Intensity is very important information for considered models from a counting point of view.

**Density**: denotes the number of vehicles on some road segment and its meaning comes near to the occupancy one, on condition of low velocities particularly. Its unit is \([\text{uv/km}]\).

**Velocity**: can be point or segmental. It determines the average speed of vehicles passing over a detector or a certain stage. Its unit is \([\text{km/h}]\).

It is necessary to measure density and velocity to determine the actual traffic situation. The standard outputs of the measurement are values of the intensity and occupancy. Experience shows that detectors are fault-prone and their reparation is far from being easy. Therefore the filtration of their data is necessary. Here, outlier filtration and normalization to zero mean and unit standard deviation were applied on the raw data.

Relations between intensity, velocity and density can be represent by graph models, for example linear model (Pict.1).

**2.2 Periodicity**

**Daily periodicity**: may be separated to daily phases. At night, the traffic intensity is very low. In the morning, the demands rapidly rise due to the cars of commuters and increased commercial transport. The intensity of the traffic reaches soon its maximum and a slight fall is observed around noon. Then, it rises again and the saturation lasts till the evening when the intensity starts to fall gradually to zero.

**Weekly periodicity**: connected with alternating work days and weekends is also strongly reflected in data.

### 2.3 Queue length

The queue represents amount of vehicles at the end of red time. The main problem is the fact that the queue length is not measurable in practice. The value of the occupancy is higher for the longer queue because the vehicles pass the detector more slowly near to the end of this queue. For the equation describing the queue length time course, the following equation is used:

\[
\xi_{t+1} = \xi_t + (I_t - P_t)T_p, \tag{1}
\]

where:
- \(\xi_t\) ... is queue length,
- \(t\) ... time instant,
- \(I_t\) ... input intensity,
- \(P_t\) ... passage (intensity on stop-line),
- \(T_p\) ... sample time.

The linear relation between the occupancy and the queue length is:

\[
O_{t+1} = \kappa \xi_t + \lambda, \tag{2}
\]

where:
- \(O_{t+1}\) ... is occupancy at time \(t + 1\),
- \(\kappa, \lambda\) ... parameters (constants).

Constants \(\kappa\) and \(\lambda\) can be determined experimentally for each set of the subsequent detectors because distances between the detectors placed on one approach specify the minimum and maximum queue lengths and corresponding limit average values of the occupancy can be measured. Assuming the linear relation, then the constants can be specified. In this paper is supposed that the parameters of occupancy dependance are known and constant in time or they can be changed continuously (known, time-dependent) or estimated (unknown, variable). For measurements mentioned above is used a pair of remote and strategic detectors. The detectors on stop-lines are not suitable (Pict.2).

### 2.4 Signal traffic control

There are three possible ways to influence the traffic flow by using the traffic signals on condition that the optimal number of stages and their constitution has been. The controlled parameters are:
**Cycle length**: time required for one complete sequence of signal displays (sum of phase green and inter-green times). For a given movement, cycle time is the sum of the durations of red, yellow and green signal displays, or sum of effective green and red times. In gap-acceptance analysis, this is the equivalent average cycle time corresponding to the block and unblock periods in the opposing traffic stream.

**Green time**: the green duration of each stage should be optimized according to the actual demand of the involved streams. The most preferably, it should be given the optimal value of the green time and the allowable interval of its change. The longer green time allows the more cars, on the corresponding approaches, to drive through the junction. On the other hand, it causes the longer waiting times for the vehicles queueing on the remainder of the approaches.

**Offset**: The specification of the offsets allows to create green waves. Ideally, it takes into account the possible existence of the queues.

3. **State space model**

The queue length and the occupancy of each junction approach are the basic state quantities for fully expressed traffic situation at given time instant. For simplicity, the state space model is derived for the junction with four arms. Each arm has only one input and output lane and all drive directions are allowed. The arms are marked by numbers from 1 to 4 anticlockwise, starting from left. The signal scheme is composed for two stages and for one input and one output detector for each arm.

3.1 **Passage and queue indicator**

The passage through the junction from the given arm depends on the actual input intensity, the actual queue length on this arm and the actual setting of the traffic lights. The structural arrangement of the junction (or the corresponding quantity of the saturation flow) and the actual control (or the relative duration of the green signal) determine the capacity of a given arm, i.e. the maximum number of vehicles that can pass safely the junction during the green light. According to initial traffic conditions are two cases. The first (3) is no queue (or it is small enough) at the beginning of the green and the input intensity is also small enough. The second (4) is fulfilled in case of the long queues or high intensities:

$$I_{i,j}Z_{i,j} + \xi_{i,j} \leq K_{i,j} = S_i Z_{i,j} \Rightarrow$$
$$\Rightarrow P_{y,j} = \alpha_y (I_{i,j}Z_{i,j} + \xi_{i,j}),$$

(3)

$$I_{i,j}Z_{i,j} + \xi_{i,j} > K_{i,j} = S_i Z_{i,j} \Rightarrow$$
$$\Rightarrow P_{y,j} = \alpha_y K_{i,j},$$

(4)

where:

- $I_{i,j}$ … is input intensity of arm $i$,
- $Z_{i,j}$ … relative green for arm $i$,
- $\xi_{i,j}$ … length of the queue on arm $i$,
- $K_{i,j}$ … capacity of arm $i$,
- $S_i$ … saturation flow of arm $i$,
- $P_{y,j}$ … passage through the junction,
- $\alpha_y$ … direction coefficient.

The queue indicator $\delta_{i,j}$ determines if there is supposed some queue on the given arm $i$ in the end of the green signal. The indicator is defined like the conventional function:
\[ \delta_{i,j} = \begin{cases} 1, & \text{when } (4) \text{ is fulfilled}, \\ 0, & \text{otherwise}. \end{cases} \tag{5} \]

The passage from the arm \( i \) to the arm \( j \) can be rewritten this way:

\[ P_{y,j} = \alpha_{y}[(1 - \delta_{j,i})(I_{i,j}z_{i,j} + \xi_{j,i}) + \delta_{j,i}K_{y,j}] \tag{6} \]

### 3.2 State equation

The state vector includes all intensities and occupancies of each input arm and it is composed arm by arm. For the mentioned junction, it is the following vector:

\[ x_{t} = [\xi_{1,i}; O_{ij}; \xi_{2,i}; O_{ij}; \xi_{3,i}; O_{ij}; \xi_{4,i}; O_{ij}]^T. \tag{7} \]

The equations (1) and (6) are used for estimating the following queue time course:

\[ \xi_{i,j+1} = \delta_{i,j} \xi_{i,j} - (1 - \delta_{i,j}) I_{i,j} + \delta_{i,j} S_{i,j} z_{i,j} + I_{i,j}, \tag{8} \]

\[ O_{i,j+1} = \kappa_{i,j} \xi_{i,j} + \beta_{i,j} O_{i,j} + \delta_{i,j}. \tag{9} \]

Applying the equations (2) and (8) to each junction arm determines the matrix state equation for the whole junction:

\[ x_{t+1} = A_{t} x_{t} + B_{t} z_{t} + f_{t} + e_{t}, \tag{10} \]

\[ A_{t} = \begin{bmatrix} \delta_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \kappa_{1,2} & \beta_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_{2,3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \kappa_{2,3} & \beta_{2,3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_{4,3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa_{4,3} & \beta_{4,3} \end{bmatrix}. \tag{11} \]

\[ B_{t} = \begin{bmatrix} b_{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b_{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{12} \]

\[ b_{j} = -(1 - \delta_{j,i}) I_{j,3} - \delta_{j,i} S_{j}, \tag{13} \]

\[ z_{t} = [z_{1,i}; z_{2,i}; z_{3,i}; z_{4,i}]^T, \tag{14} \]

\[ f_{t} = [I_{1,j}; I_{2,j}; I_{3,j}; I_{4,j}; I_{5,j}; I_{6,j}; I_{7,j}; I_{8,j}]^T, \tag{15} \]

where:

\[ x_{t} \ldots \text{is state vector}, \]

\[ A_{t} \ldots \text{matrix of time variant of parameters}, \]

\[ B_{t} \ldots \text{matrix of time variant matrix passages}, \]

\[ z_{t} \ldots \text{vector of the relative greens}, \]

\[ f_{t} \ldots \text{vector of time variant constants}, \]

\[ e_{t} \ldots \text{vector of random process noise}. \]

### 3.3 Output equation

For maximum utilizing actual traffic data and correcting the state estimates is used Kalman filter (Peterka, 1981). First, it is necessary to select all quantities which can be measured and compared to predicted ones. Thus it can be used only occupancies of input detectors and intensities of output detectors for correcting the whole state vector. Supposing the input and output detector for each junction arm, the modeled output is:

\[ y_{i,t} = [y_{1,i}; y_{2,i}; y_{3,i}; y_{4,i}; y_{5,i}; O_{1,i}; O_{2,i}; O_{3,i}; O_{4,i}]^T, \tag{16} \]

where:

\[ y_{i,t} \ldots \text{is measured output of arm } i, \]

\[ O_{i,t} \ldots \text{time variant matrix of parameters}. \]

First is determined the total outputs (the passages) into the arm \( i \), i.e. the sum of the individual arm passages that head towards the given arm \( i \). We use the knowledge of the direction coefficients (the ratios of the turning vehicles).

According to the relation (6), the arm output like the sum of the passages from the remaining arms is:

\[ y_{i,t} = \sum_{j} \alpha_{ij} [I_{i,j} \xi_{j,i} + I_{i,j} z_{j,i} + \delta_{j,i} S_{j} z_{j,i}], \tag{17} \]

\[ \sum_{j} \alpha_{ij} = 1 \land \alpha_{ii} = 0, \forall i = 1, 2, 3, 4. \tag{18} \]
For determining the matrix output equation, the modified equation (17) and the identity equation of the occupancy:

\[ y_{ij} = \sum_j [\alpha_j \left[ (1 - \delta_{ij}) \xi_{ij} + \left( 1 - \delta_{ij} \right) I_{ij} + \delta_{ij} S_{ij} \right] ] \]

(19)

applied for each arm with the input and output detector. The matrix output equation then is:

\[ y_t = C_t x_t + D_t z_t + \epsilon_t, \]  

(20)

\[
C_t = \begin{bmatrix}
0 & 0 & c_{21} & 0 & c_{31} & 0 & c_{41} & 0 \\
c_{12} & 0 & 0 & 0 & c_{12} & 0 & c_{42} & 0 \\
c_{13} & 0 & c_{23} & 0 & 0 & c_{43} & 0 & 0 \\
c_{14} & 0 & c_{24} & 0 & c_{34} & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix},
\]

(21)

\[ c_{ij} = \alpha_{ij} (1 - \delta_{ij}), \]  

(22)

\[
D_t = \begin{bmatrix}
0 & d_{21} & d_{31} & d_{41} \\
d_{12} & 0 & d_{32} & d_{42} \\
d_{13} & 0 & d_{23} & 0 & d_{43} \\
d_{14} & d_{24} & d_{34} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

(23)

\[ d_{ij} = \alpha_{ij} (1 - \delta_{ij}) I_{ij} - \delta_{ij} S_{ij}. \]  

(24)

where:

- \( x_t \) … is state vector,
- \( C_t \) … time variant matrix of parameters,
- \( D_t \) … time variant matrix of passages,
- \( z_t \) … vector of the relative greens,
- \( \epsilon_t \) … measurement noise.

All matrices of the mentioned state space model are supposed to be automatically generated for any set of junctions on the basis of traffic data measurement at disposal.

4. Conclusion

In the Institute of Information Theory and Automation of the Academy of Science of the Czech Republic is developed (by J. Homolová, I. Nagy, etc.) an algorithm which estimates parameters of this model. The algorithm is tested on real traffic samples.

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References


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Ing. Pavel DOHNAL was born in 1980. He graduated CTU FEE Prague in 2003 the bachelor profile Cybernetics and Measurement and in 2005 the master profile Technical Cybernetics. Actually, he studies the doctoral program: Electrical Engineering and Information Technology on CTU FEE, profile Control Engineering and Robotics and does research on Academy of Sciences of the Czech Republic.