

A New Approach to Estimating the Bellman Function

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Dynamic programming

System - part of world interesting for decision maker

Decision maker - human or machine with **aims** according to system

Decisions x_t is designed by decision maker to reach the aims

Output y_t is information about system available to decision maker

Gain function - degree of reaching the aims

$$G : (x_1, \dots, x_T, y_1, \dots, y_T) \rightarrow \mathcal{R}_0^+$$

The main aim - search the sequence $\{x_1, \dots, x_T\}$ to maximize:

$$\max_{\{x_1, \dots, x_T\}} G = \max_{\{x_1, \dots, x_T\}} \sum_{k=1}^T g_k.$$

Dynamic programming - Bellman function

Off-line optimization at the time t :

$$\mathcal{V}_t = \max_{\{x_t, \dots, x_T\}} \sum_{k=t}^T g_k$$

Bellman function (recursive shape):

$$\mathcal{V}_t = \max_{x_t} (g_t + \mathcal{V}_{t+1})$$

Optimal decision:

$$x_t = \arg \max_{x_t} (g_t + \mathcal{V}_{t+1})$$

Dynamic programming - Bellman function

On-line optimization at the time t :

$$\mathcal{V}_t = \max_{\{x_t, \dots, x_T\}} E \left(\sum_{k=t}^T g_k \mid x_1, \dots, x_{t-1}, y_1, \dots, y_t \right)$$

Bellman function (recursive shape):

$$\mathcal{V}_t = \max_{x_t} E (g_t + \mathcal{V}_{t+1} \mid x_1, \dots, x_{t-1}, y_1, \dots, y_t)$$

Optimal decision:

$$x_t = \arg \max_{x_t} E (g_t + \mathcal{V}_{t+1} \mid x_1, \dots, x_{t-1}, y_1, \dots, y_t)$$

Off-line strategy analysis

Assumption

- The decision x_t does not influence the system.

The optimal strategy obtained at whole dataset:

$$X^T = (x_1^T, x_2^T, \dots, x_T^T)$$

Optimal strategies for shorter horizon:

$$X^1 = (x_1^1)$$

$$X^2 = (x_1^2, x_2^2)$$

$$X^3 = (x_1^3, x_2^3, x_3^3)$$

$$\vdots$$

$$X^T = (x_1^T, x_2^T, x_3^T, x_4^T, \dots, x_T^T)$$

Similarity indexes

Similarity index (number of similar decisions):

$$S_t = \sum_{i=1}^t \delta(x_i^t, x_i^T)$$

Strict similarity index (length of non-broken similarity):

$$s_t = \max_i \{i; (\forall j \in \mathcal{N})(j \leq i \Rightarrow x_j^t = x_j^T)\}$$

$$s_t \leq S_t \leq t$$

Example:

$$\begin{array}{r}
 X^t = \\
 X^T =
 \end{array}
 \begin{array}{cccccccc}
 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \dots t \\
 \{ 1 & -1 & 0 & 1 & 0 & 1 & 0 & 0 \dots 0 \} \\
 \{ 1 & -1 & 0 & 1 & 1 & 1 & 0 & 1 \dots 1 \dots \}
 \end{array}$$

$$s_t = 4, \quad S_t = 6$$

Estimating the Bellman function

Bellman function (recursive shape):

$$\mathcal{V}_t = \max_{x_t} E(g_t + \mathcal{V}_{t+1} | x_1, \dots, x_{t-1}, y_1, \dots, y_t)$$

If $s_t \approx t$ and we insert the X^t , we obtain system of functional equations:

$$\mathcal{V}_k = \max_{x_k} E(g_k + \mathcal{V}_{k+1} | x_1^t, \dots, x_{k-1}^t, y_1, \dots, y_k) \quad \text{for } k \in \{1, \dots, s_t\},$$

which can be transformed to system algebraic equations for parametrized shape of Bellman function.

Solution is inserted into on-line Bellman equation and the maximization can be calculated.

Futures trading: task definition

Futures futures contract, obligation to buy a normalized amount of a commodity

Speculator chooses the decision about the future

Long - believe in price increase

Short - believe in price decrease

Flat - do not believe - out of market

Gain function:

$$G = \sum_{k=1}^T \underbrace{(y_k - y_{k-1})x_{k-1} - C|x_{k-1} - x_k|}_{g_k},$$

where y_1, \dots, y_T price of contract
 x_1, \dots, x_T count of held contracts
 C transaction cost per contract

Calculated variables

Five reference price sequences:

- Cocoa (CC)
- Light crude oil (CL)
- U.S. Treasury note (FV2)
- Japanese Yen (JY)
- Wheat (W)

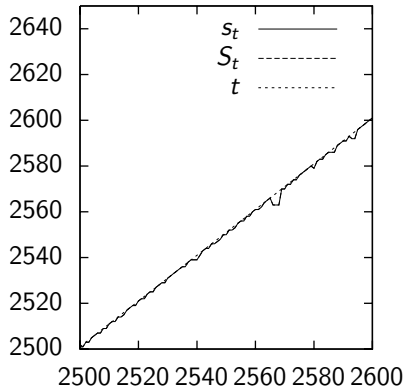
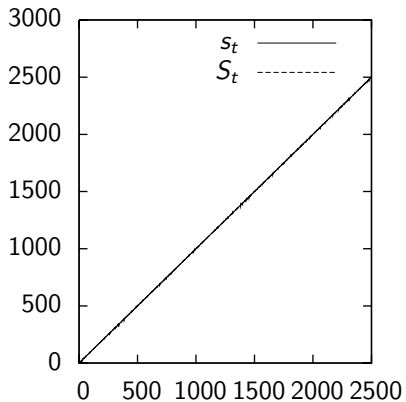
Calculated variables:

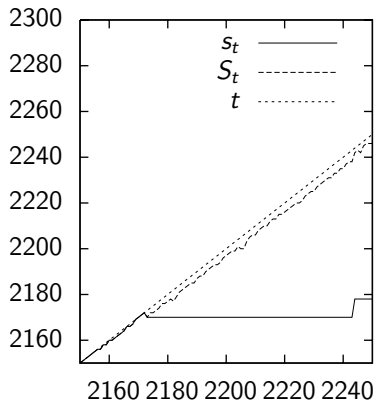
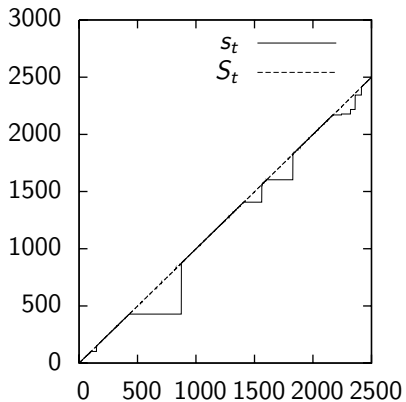
- $c_1 = \max_t(t - s_t)$
- $c_2 = \max_t(t - S_t)$
- $t_{ch,1}$ and $t_{ch,2}$ - last change of value c_1 and c_2

Table of results

Market	c_1	c_2	$t_{ch;1}$	$t_{ch;2}$	T
CC	6	6	342	342	3822
CL	444	6	847	2205	3863
FV2	8	8	383	383	3766
JY	4	4	50	50	3871
W	7	7	2452	2452	3822

Table: Dominating constants c_1 and c_2





Experiment setup

Experiment:

- Parametrized shape of Bellman function
- Weighted least squares method
- Prediction using autoregressive model

⇒ iteration spread in time (IST)

Reference results:

- Model predictive method (MPC)
- Predictive model and task setup were same as above

Results

Market	MPC	IST
CC	-6 450	-1 490
CL	-12 350	3 390
FV2	-5 701	10 727
JY	-26 568	-35 247
W	-9 792	-1 923

Table: Results of experiment

Conclusion

- IST is better in 4 of 5 datasets.
- The approach needs further testing.
- The similarity indexes are calculated non-causally.
- The causal criterion of usage the approach should be find.