COMPARISON OF MODELS FOR RESULTS OF SPORT MATCHES

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1 MODELS OF SCORE OF SPORT MATCHES

1. *Independent Poissons*

In a match between teams i (home) and j (away), let X_{ij} and Y_{ij} be the number of goals scored by them. Then

$$X_{ij} \sim \text{Poisson}\left(\log\lambda_1 = \alpha_i + \beta_j + \gamma\right), \qquad Y_{ij} \sim \text{Poisson}\left(\log\lambda_2 = \alpha_j + \beta_i\right), \tag{1}$$

X_{ij} and Y_{ij} are independent, α_i = `attack' rate of the team $i$, $\beta_i$ =`defense' rate,

$\gamma > 0$ quantifying the effect of the home field

Improvements of model (1) - as certain results appear more or less frequently than predicted by (1):

2. *Inflated Poisson model*

- the mixture of (1) and certain discrete probability on those results:

$$P\left(X_{ij} = x, Y_{ij} = y\right) = (1-p).\{\, Model\ (1)\} + p.P_{\mathrm{D}}(x,y)$$

3. *Bivariate Poisson model*

Let Y_0, Y_1, Y_2 be independent Poisson variables with parameters $\lambda_i, i = 0,1,2,$ set $X = Y_1 + Y_0, Y = Y_2 + Y_0$.
Then the random vector (X,Y) has the **bivariate Poisson distribution** with probability

$$P(X = x, Y = y) = e^{-(\lambda_1+\lambda_2+\lambda_0)} \frac{\lambda_1^{\ x} \lambda_2^{\ y}}{x!\,y!} \cdot \sum_{i=0}^{\min(x,y)} \binom{x}{i}\binom{y}{i} i! \left(\frac{\lambda_0}{\lambda_1\lambda_2}\right)^i .$$

The marginal distribution of X is again the Poisson $(\lambda_1 + \lambda_0)$ one, similarly for Y, and $\lambda_0 = \mathrm{cov}\,(X, Y)$. Again

$$\log \lambda_{1ij} = \delta + \gamma + \alpha_i + \beta_j, \quad \log \lambda_{2ij} = \delta + \gamma + \alpha_j + \beta_i, \quad \log \lambda_{0ij} = \beta_0,$$

describe the match of teams i, j with i at its home field, δ - could be another covariate indicating certain match situation.

4. *Inflated bivariate Poisson model*

$$p(x, y) = (1 - p) \cdot BP(x, y, \lambda_0, \lambda_1, \lambda_2) + p \cdot P_D(x, y),$$

2 SOME RESULTS WITH ‚STATIC‘ MODELS

- comparison of real results and the prediction of victories at home, away, and of draws, for different models of Part 1, based on data from 1998 – 2001. The 'Param' is the number of model parameters (one attack parameter is set to 0,for the sake of uniqueness)

Model type	Remark	P-home	Draw	P-away	Param.
Real values		0.4563	0.2745	0.2692	
Independent Poissons		0.4618	0.2521	0.2861	184
Diag.Infl.Ind.Poissons	D=Discrete	0.4509	0.2725	0.2766	187
Bivariate Poisson	λ_0 =const.	0.4592	0.2611	0.2797	185
Diag.Infl.Biv.Poisson	D=Discrete	0.4513	0.2752	0.2735	188

Table 1. Comparison of models for the English Premier League, Divisions 1-3 and FA Cup (92 teams).

Model type	Remark	P-home	Draw	P-away	Param.
Real values		0.4343	0.2168	0.3489	
Independent Poissons		0.4524	0.1711	0.3765	60
Diag.Infl.Ind.Poissons	D=Discrete	0.4360	0.2029	0.3611	63
Bivariate Poisson	λ_0 =const.	0.4517	0.1735	0.3748	61
Diag.Infl.Biv.Poisson	D=Discrete	0.4364	0.2028	0.3608	64

Table 2. Comparison of models for the National Hockey League (30 teams).

3 MODELS WITH TIME-DEPENDENT PARAMETERS

- updating, temporal development of parameters
- actually, an implicit updating follows from re-computing the parameters after each week results

3.1 Weighted history models

gives larger weight to recent results than to older ones, [2]. In the framework of the GLIM, we can write the likelihood as

$$L = \prod_i \left\{ \exp\left(\frac{y_i \cdot \theta - b(\theta)}{a(\phi)} - c(y_i, \phi) \right) \right\}^{\psi(t - t_i)} ,$$

t is the actual time, t_i is the time of ***i***-th match, ψ is the weight function, non-increasing. The simplest choices:

- threshold function, $\psi(z) = 1$ for $z < T$ and $= 0$ otherwise,
- ***exponentially decreasing function*** $\psi(z) = \exp(-\nu z)$:

  When ν is settled, the analysis uses weighted MLE,

  the search for optimal ν computationally involved, even in independent Poisson model

The optimal values found were $\nu = 0.00275$ for the English football results (we analyzed together the results of 92 teams from the premier League, Division 1-3 and also participating in the FA Cup, in years 1998 – 2002),

$\nu = 0.00375$ for the ice-hockey NHL, from 30 teams and the same years . The time is in days.

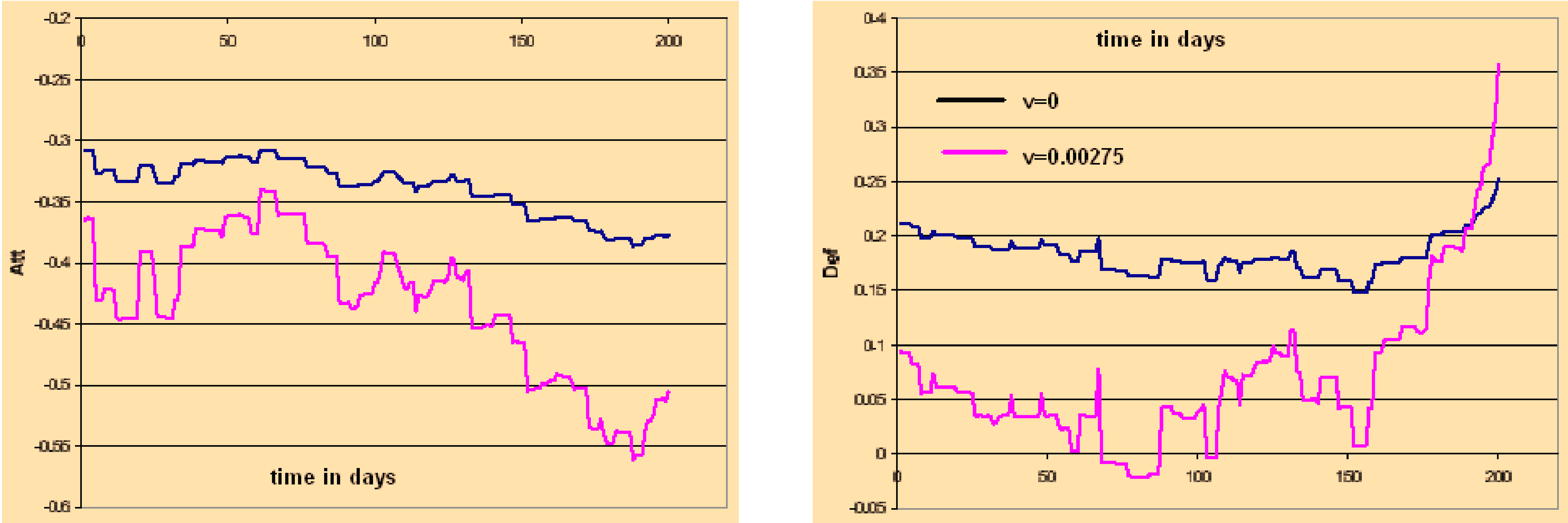

Figure 1. Development of attack parameter (left) and defense parameter (right) of Aston Villa in season 2001-2002 of English Premiér League, by metod of weighting, when parameter was selected v=0 and v=0.00275.

3.2 *Dynamic model of parameters development*

Assume a model of parameters development, $\beta_t = G_t \cdot \beta_{t-1} + \omega(t)$,

and observed variables $Y_t \sim p_t(y, \beta_t)$.

In Bayes setting, the first equation specifies the prior distribution of parameters, in fully Gauss case the scheme could be regarded also as the Kalman filter.

The objective is to find the maximizers of the posterior distributions $p(\beta_t | Y_1, \ldots, Y_t)$ (Bayes estimate),

and predictive distribution $p(Y_{t+1} | D_t)$, where D_t ~ information available at time ***t***.

A variant = the approach based on the MCMC (which as a rule uses long computations too, but repeates rather simple steps)

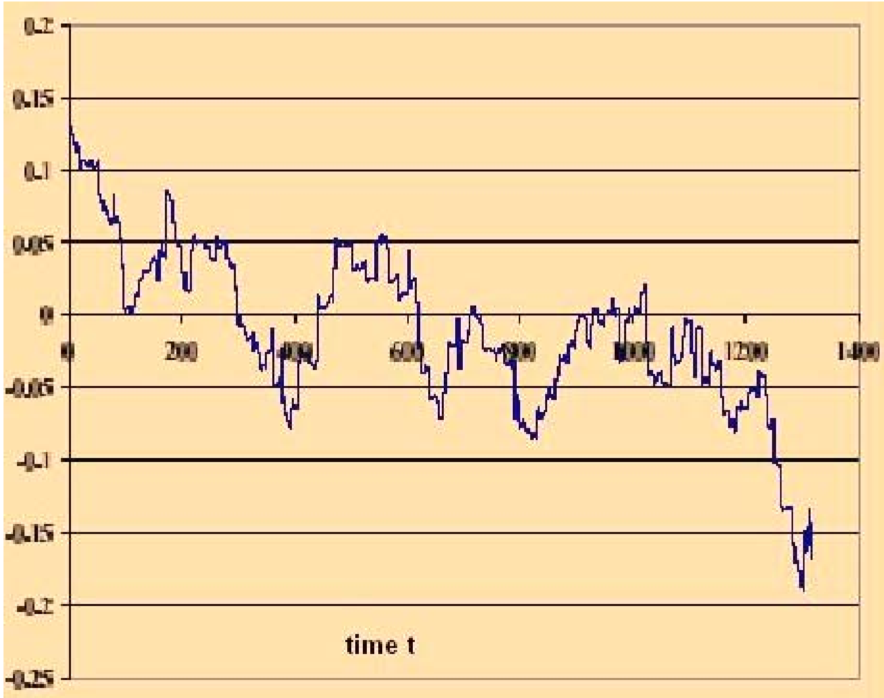

Figure 2. Development of attack parameter (left) and defense parameter (right) of Tampa Bay (NHL)in season 2003-2004 (whenT.B. was the winner of the Stanley Cup) computed from dynamic GLIM.
Time=order of all NHL matches this season.

4 SELECTED RESULTS WITH DYNAMIC MODELS

Tables compare the results from weighted models, Part 3.1, and dynamic GLIM (Part 3.2) for the NHL. Starting estimates of parameters and distributions were obtained from years 1998 – 2001, then the dynamic models were updated and results predicted (one-round forward) for next 4 seasons 2001 – 2005.

We included also the trinomial logistic regression model modelling just probabilities of home win, away win, draw as a categorial variable, with parameters updated as in Part 3.1.

Model type	Remark	P-home	Draw	P-away
Real values		0.4547	0.2699	0.2754
Independent Poissons		0.4615	0.2545	0.2839
Weighted Ind.Poissons	$e^{-\nu t}$	0.4597	0.2530	0.2873
W. Diag.Infl.Biv.Poisson	λ_0 const, Discrete	0.4466	0.2804	0.2730
Weighted trinomial		0.4554	0.2745	0.2701

Table 3. Comparison of predicted and real numbers of different results for English Pemier League, Divisions 1-3 and FA Cup in seasons 2001 – 2005.

Model type	Remark	P-home	Draw	P-away
Real values		0.4267	0.2400	0.3333
Independent Poissons		0.4566	0.1708	0.3726
Weighted Ind.Poissons	$e^{-\nu t}$	0.4600	0.1717	0.3683
W. Diag.Infl.Biv.Poisson	λ_0 const, Discrete	0.4393	0.2079	0.3528
Weighted trinomial		0.4283	0.2368	0.3349
Dynamic GLIM		0.4571	0.1735	0.3694

Table 4. Comparison of predicted and real results in NHL 2001 – 2005.

From those ‚average‘ numbers of results, it seems that the results closest to real ones are obtained from the simplest, trinomial model. However, it does not mean that it predicts best the results of individual matches.

***‚Simulated‘ betting**:* We compared predictive ability of models and resulting return of investment (ROI). We used the Kelly 1/10 method of investment, in cases when overlay was > 0. Results are on following graphs.

Remark: If official course of certain result is k and the model gives for the same result probability *p*,

the ***overlay*** $o = k.p - 1$ measures the diference (and advantage, if $o>0$ and the model is a good one).

Then ***Kelly‘s strategy*** recommends to bet $K = 100 \cdot \frac{o}{k-1}$ (%) part of account available. Fractional Kelly (e.g.1/10 Kelly) then means $K/10$.

On the following graphs, notice the decrease of prediction effectivness in the beginning of all seasons, caused evidently by the changes of teams which are not anticipated and must be learned by parameters updating.

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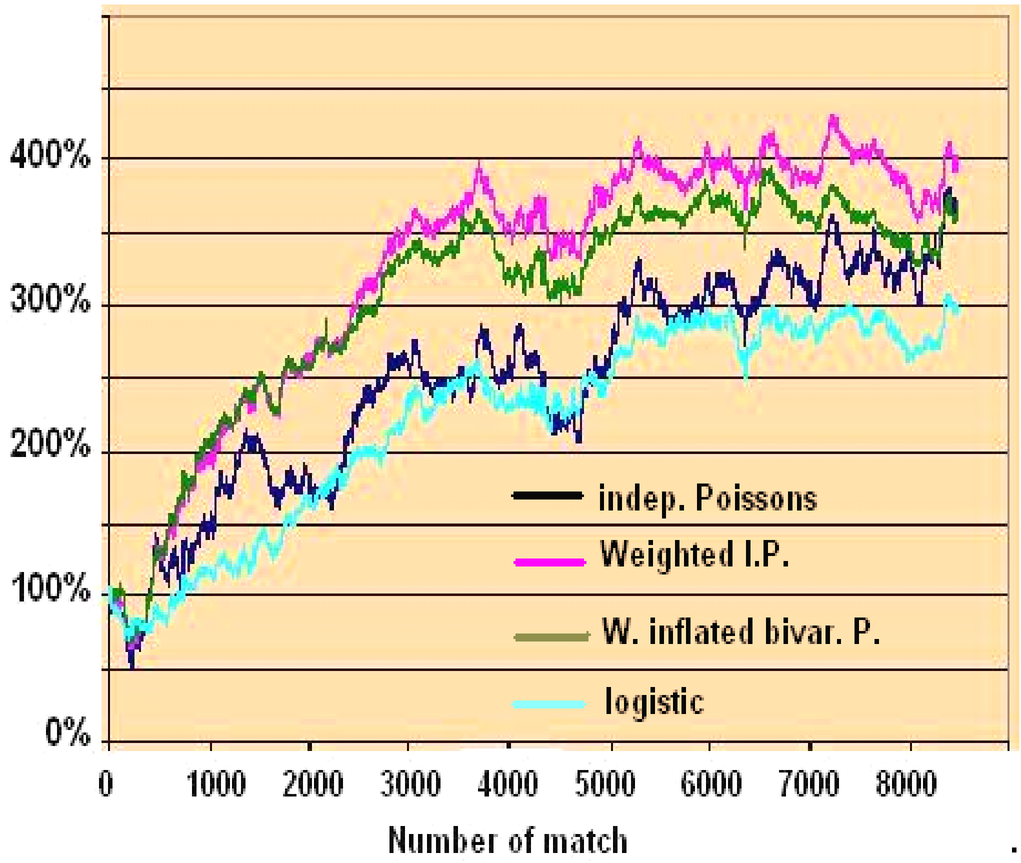

Figure 3. Development of account (in %) under 1/10 Kelly betting method, based on different models for English football 2001 – 2005.

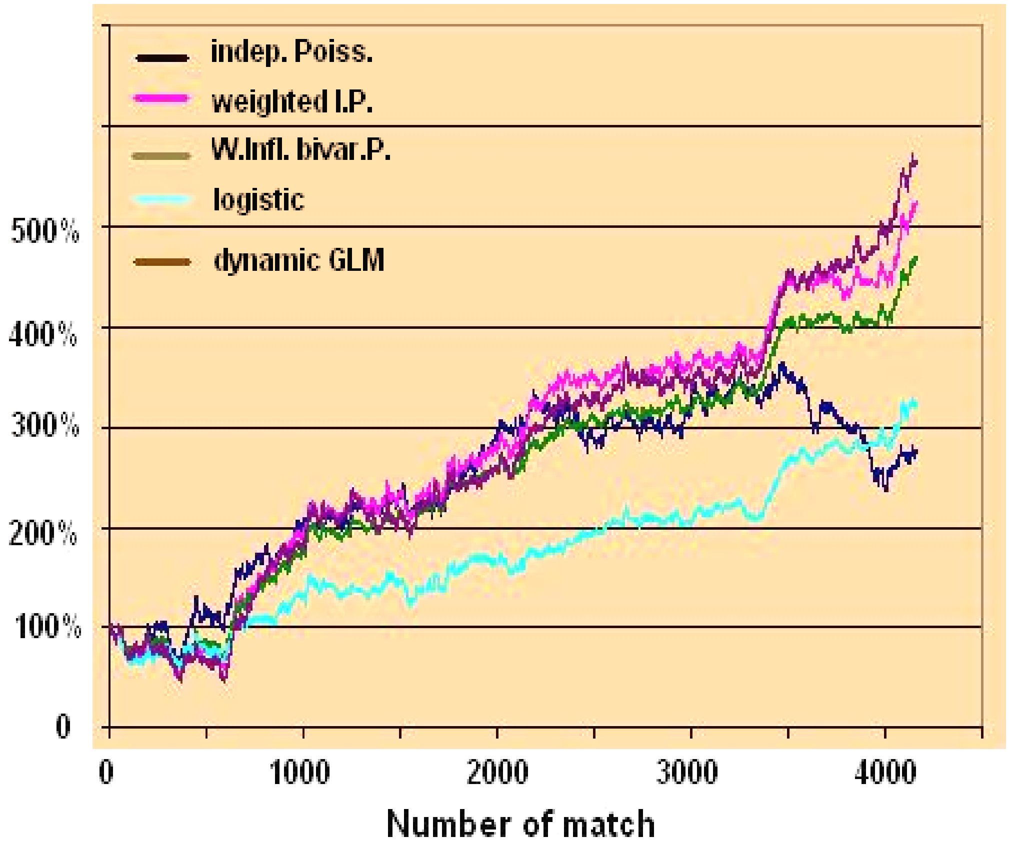

Figure 4. Development of account (in %) under 1/10 Kelly betting method, based on different models for NHL 2001 – 2005.

Next 2 graphs show how different betting systems can be successful (and also unstable):

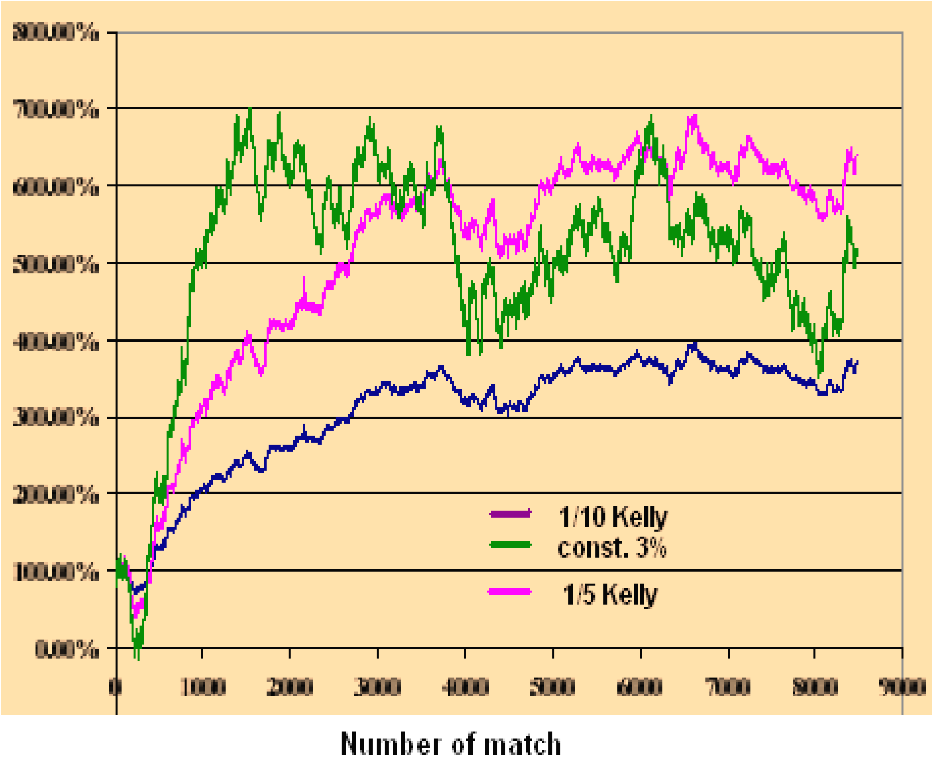

Figure 5. Development of account (in %) under 3 different betting methods and weighted independent Poisson models for English football 2001 – 2005

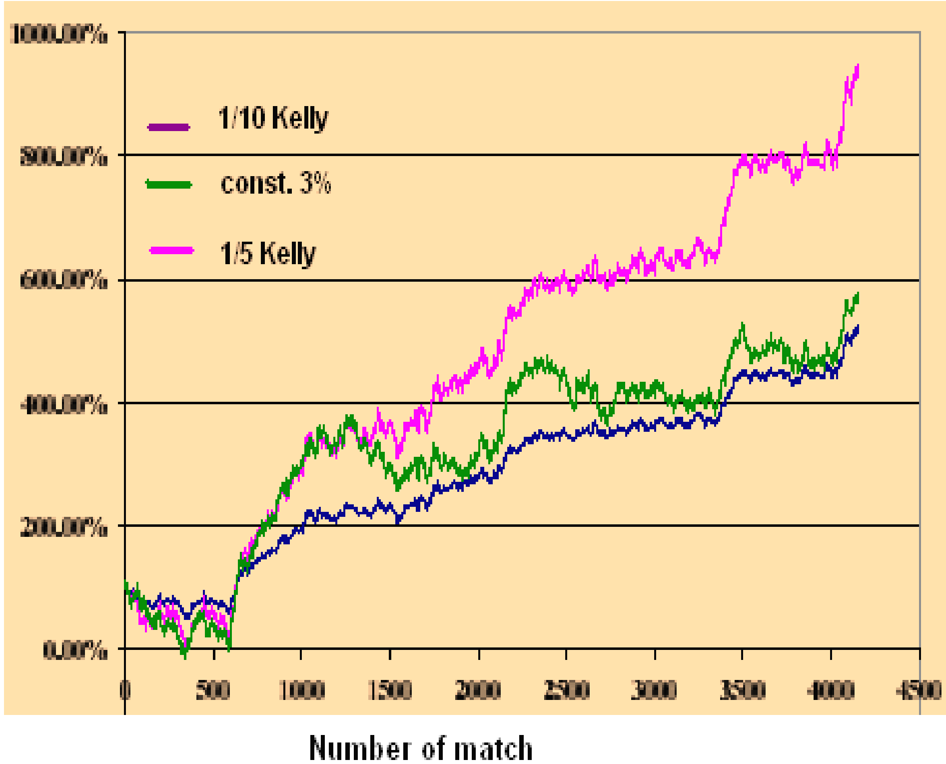

Figure 6. Development of account (in %) under 3 different betting methods and weighted independent Poisson models for NHL2001 – 2005.

Figures 7- 8. Some results for other European football leagues:

Season 2003-2004, indep. Poisson model learned from preceding season, 4 different betting methods (Kelly, ½ Kelly, ¼ Kelly, constant % from account), for the Czech Extra-league (left), Italian Serie A (right). On X-axis is the number of match, on Y-axis the account development: