Multisensor Information Fusion

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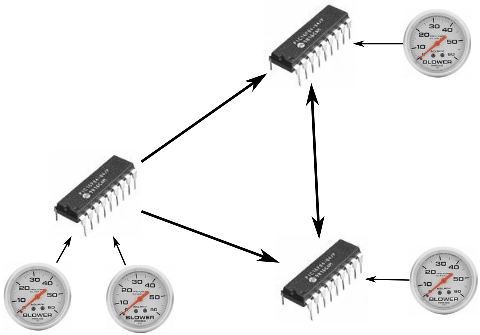
Outline

Multisensor System

sensor ($\triangleq$ estimator + measurement device) + communication

- when it is better to communicate estimates than measurements (e.g. radar tracking, traffic information, ...)



- each estimator has its own measurement devices
- no knowledge of remote measurement equation required
- only estimates communicated among estimators

Multisensor System

System Description

The system is linear Gaussian given by

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{F}_k \mathbf{x}_k + \mathbf{w}_k, \\ \mathbf{z}_k &= \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k, \end{aligned}$$

with zero-mean independent white noises, $\mathrm{cov}(\mathbf{w}_k) = \mathbf{Q}_k$, $\mathrm{cov}(\mathbf{v}_k) = \mathbf{R}_k$, etc.

OR

The system is generally given by density functions

$$p(\mathbf{x}_{k+1}|\mathbf{x}_k),$$
$$p(\mathbf{z}_k|\mathbf{x}_k).$$

Multisensor System

"global" measurement $\mathbf{z}_k$ consists of N local measurements $\mathbf{z}_k^{(i)}$

Measurement Equation Partition

linear system

$$\begin{bmatrix} \mathbf{z}_k^{(1)} \\ \mathbf{z}_k^{(2)} \\ \vdots \\ \mathbf{z}_k^{(N)} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_k^{(1)} \\ \mathbf{H}_k^{(2)} \\ \vdots \\ \mathbf{H}_k^{(N)} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{v}_k^{(1)} \\ \mathbf{v}_k^{(2)} \\ \vdots \\ \mathbf{v}_k^{(N)} \end{bmatrix}$$

system given by pdfs

$$p(\mathbf{z}_k^{(1)}, \mathbf{z}_k^{(2)}, \ldots, \mathbf{z}_k^{(N)} | \mathbf{x}_k)$$

Multisensor System

it is desirable to have local measurements independent on remote measurements

Spatially Independent Measurements

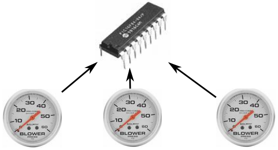
$$\mathrm{E}[\mathbf{v}_k \mathbf{v}_k^T] = \begin{bmatrix} \mathbf{R}_k^{(11)} & \cdots & \mathbf{R}_k^{(1N)} \\ \vdots & \ddots & \vdots \\ \left(\mathbf{R}_k^{(1N)}\right)^T & \cdots & \mathbf{R}_k^{(NN)} \end{bmatrix} \overset{sp.ind.}{=} \begin{bmatrix} \mathbf{R}_k^{(11)} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{R}_k^{(NN)} \end{bmatrix}$$

$$p(\mathbf{z}_k^{(1)}, \mathbf{z}_k^{(2)}, \ldots, \mathbf{z}_k^{(N)} | \mathbf{x}_k) \overset{sp.ind.}{=} \prod_{j=1}^{N} p(\mathbf{z}_k^{(j)} | \mathbf{x}_k)$$

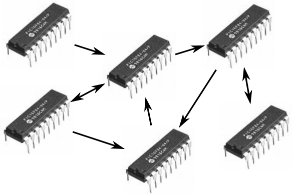
Estimators Network Architecture

communication rules the information fusion

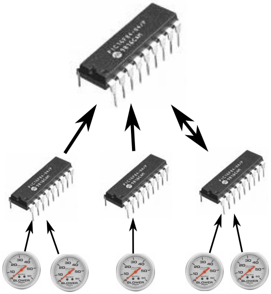
Centralised Architecture



Decentralised Architecture



Hierarchical Architecture



Centralised Kalman Filter

MSE optimal estimate, one estimator

State Space Kalman Filter Equations

filtering equation

$$\begin{aligned} \mathbf{P}_{k|k}^{-1}\hat{\mathbf{x}}_{k|k} &= \mathbf{P}_{k|k-1}^{-1}\hat{\mathbf{x}}_{k|k-1} + \boxed{\mathbf{H}_k^T\mathbf{R}_k^{-1}\mathbf{z}_k}, \\ \mathbf{P}_{k|k}^{-1} &= \mathbf{P}_{k|k-1}^{-1} + \boxed{\mathbf{H}_k^T\mathbf{R}_k^{-1}\mathbf{H}_k}, \end{aligned}$$

update equation

$$\begin{aligned} \hat{\mathbf{x}}_{k|k-1} &= \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1|k-1} \\ \mathbf{P}_{k|k-1} &= \mathbf{F}_{k-1}\mathbf{P}_{k|k-1}\mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1} \end{aligned}$$

Centralised Kalman Filter

Information Space Kalman Filter Filtering Equation

by defining $\hat{\mathbf{y}}_{k|j} \triangleq \mathbf{P}_{k|j}^{-1}\hat{\mathbf{x}}_{k|j}$ (information),
$\mathbf{Y}_{k|j} \triangleq \mathbf{P}_{k|j}^{-1}$ (information matrix):

$$
\begin{aligned}
\hat{\mathbf{y}}_{k|k} &= \hat{\mathbf{y}}_{k|k-1} + \mathbf{i}_k, \\
\mathbf{Y}_{k|k} &= \mathbf{Y}_{k|k-1} + \mathbf{I}_k,
\end{aligned}
$$

where $\mathbf{i}_k \triangleq \mathbf{H}_k^T\mathbf{R}_k^{-1}\mathbf{z}_k$ and $\mathbf{I}_k \triangleq \mathbf{H}_k^T\mathbf{R}_k^{-1}\mathbf{H}_k$

Posterior Probability Equation

$$
p(\mathbf{x}_k|\mathbf{z}_k, \mathbf{Z}^{k-1}) \propto p(\mathbf{z}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{Z}^{k-1})
$$

Hierarchical Combination of Estimates

Estimate Error Is a Measurement Error

$$\hat{\mathbf{x}}_{k|k}^{(j)} = \mathbf{x}_k + (\hat{\mathbf{x}}_{k|k}^{(j)} - \mathbf{x}_k) = \mathbf{x}_k + (-\tilde{\mathbf{x}}_{k|k}^{(j)})$$

Fusion Centre Measurement Equation

$$\begin{bmatrix} \hat{\mathbf{x}}_{k|k}^{(1)} \\ \vdots \\ \hat{\mathbf{x}}_{k|k}^{(N)} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{n_x} \\ \vdots \\ \mathbf{I}_{n_x} \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} -\tilde{\mathbf{x}}_{k|k}^{(1)} \\ \vdots \\ -\tilde{\mathbf{x}}_{k|k}^{(N)} \end{bmatrix}$$

or briefly

$$\mathbf{z}_k^{FC} = \mathbb{I}_N \mathbf{x}_k + \xi_k, \quad \mathrm{cov}(\xi_{\mathbf{k}}) = \mathbf{P}_k = [\mathbf{P}_{k|k}^{(ij)}]_{i,j=1}^{N}$$

Maximum Likelihood Estimate

Cross-Covariances of Estimate Errors

$$\mathbf{P}_{k|k}^{(ij)} = \mathrm{E}(\tilde{\mathbf{x}}_{k|k}^{(i)} \tilde{\mathbf{x}}_{k|k}^{(j)T}) \overset{i \neq j}{=}$$
$$(\mathbf{I}_{n_x} - \mathbf{K}_k^{(i)} \mathbf{H}_k^{(i)})(\mathbf{F}_{k-1} \mathbf{P}_{k-1|k-1}^{(ij)} \mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1})(\mathbf{I}_{n_x} - \mathbf{K}_k^{(j)} \mathbf{H}_k^{(j)})^T$$

$\mathbf{K}_k^{(i)}$, $\mathbf{K}_k^{(j)}$ transmitted to the centre = communication burden

Time Dependence of Estimate Errors

the noise of $\{\mathbf{z}_k^{FC}\}$ is NOT white

$$\mathrm{E}(\tilde{\mathbf{x}}_{k|k}^{(j)} \tilde{\mathbf{x}}_{k-1|k-1}^{(j)T}) = (\mathbf{I}_{n_x} - \mathbf{K}_k^{(j)} \mathbf{H}_k^{(j)}) \mathbf{F}_{k-1} \mathbf{P}_{k-1|k-1}^{(jj)} \neq 0$$

Maximum Likelihood Estimate

Fusion Centre Measurement Equation

$$\mathbf{z}_k^{FC} = \mathbb{I}_N \mathbf{x}_k + \xi_k$$

BUT

$\{\xi_k\}$ is a COLOURED noise
and
ξ_k and $\mathbf{x}_k$ are CORRELATED

Fusion Centre Estimate and Estimate Error Covariance

$$\begin{aligned} \hat{\mathbf{x}}_{k|k} &= (\mathbb{I}_N^T \mathbf{P}_k^{-1} \mathbb{I}_N)^{-1} \mathbb{I}^T \mathbf{P}_k^{-1} \mathbf{z}_k^{FC}, \\ \mathbf{P}_{k|k} &= (\mathbb{I}_N^T \mathbf{P}_k^{-1} \mathbb{I}_N)^{-1} \end{aligned}$$

Fusion of Measurement Information

fusion of dependent information complicated
⇒ information decorrelation

Product of Block/ Block-Diagonal Matrices

IF local measurement noises are independent
THEN

$$\mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{H}_k = \sum_{j=1}^{N} \mathbf{H}_k^{(j)\,T} \mathbf{R}_k^{(j)\,-1} \mathbf{H}_k^{(j)},$$

$$\mathbf{H}_k^T \mathbf{R}_k^{-1} \mathbf{z}_k = \sum_{j=1}^{N} \mathbf{H}_k^{(j)\,T} \mathbf{R}_k^{(j)\,-1} \mathbf{z}_k^{(j)}$$

Fusion of Measurement Information

Sum of Information/ Information Matrices

the previous equations written in information space:

$$\mathbf{i}_k = \sum_{i=1}^{N} \mathbf{i}_k^{(j)},$$

$$\mathbf{I}_k = \sum_{i=1}^{N} \mathbf{I}_k^{(j)}$$

$\Rightarrow$ independent information can be summed

Distributed Kalman Filter

DKF update equations

$$\mathbf{P}_{k|k}^{-1}\hat{\mathbf{x}}_{k|k} = \mathbf{P}_{k|k-1}^{-1}\hat{\mathbf{x}}_{k|k-1} + \sum_{j=1}^{N}\left(\mathbf{P}_{k|k}^{(j)}{}^{-1}\hat{\mathbf{x}}_{k|k}^{(j)} - \mathbf{P}_{k|k-1}^{(j)}{}^{-1}\hat{\mathbf{x}}_{k|k-1}^{(j)}\right)$$

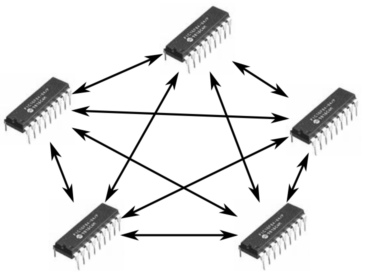
$$\mathbf{P}_{k|k}^{-1} = \mathbf{P}_{k|k-1}^{-1} + \sum_{j=1}^{N}\left(\mathbf{P}_{k|k}^{(j)}{}^{-1} - \mathbf{P}_{k|k-1}^{(j)}{}^{-1}\right)$$

(global estimate feed back to the local estimators)

pdf's equations

$$p(\mathbf{x}_k|\mathbf{z}_k, \mathbf{Z}^{k-1}) \propto p(\mathbf{x}_k|\mathbf{Z}^{k-1}) \prod_{j=1}^{N} \frac{p(\mathbf{x}_k|\mathbf{z}_k^{(j)}, \mathbf{Z}^{k-1})}{p(\mathbf{x}_k|\mathbf{Z}^{k-1})}$$

Intractable Decentralisation



simple decentralisation

=

each estimator is central

BUT

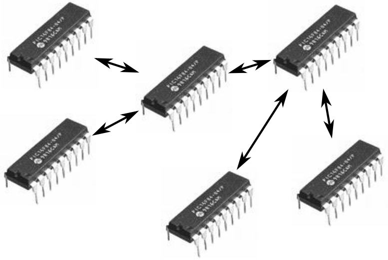
too much communication

Channel Filters

new information = incoming information – common information

multipath propagation:

A passes new information to B, B passes it further to C,
then A passes it directly to C $\rightarrow$ not new again!



IDEA:

- tree structure - one path
- calculating common information

Channel Filters

Conditioning on Two Data Sets

$$p(\mathbf{x}_k|\mathcal{Z}_A \cup \mathcal{Z}_B) = \frac{p(\mathbf{x}_k|\mathcal{Z}_A)p(\mathbf{x}_k|\mathcal{Z}_B)}{p(\mathbf{x}_k|\mathcal{Z}_A \cap \mathcal{Z}_B)}$$

$\mathcal{Z}^i_{k^-} = \mathcal{Z}^i_{k-1} \cup \mathbf{z}^{(i)}_k \triangleq$ set of measurements used by i^{th} estimator after filtering but before fusion at time k

$\mathcal{Z}^i_k = \mathcal{Z}^i_{k-1} \cup \mathbf{z}^{(i)}_k \bigcup_{j \in \mathcal{N}_i} \mathcal{Z}^j_{k^-} \triangleq \ldots$ after fusion at time k

$\mathcal{N}_i =$ neighbours of i

Prediction Step

$$p(\mathbf{x}_k|\mathcal{Z}_{k-1}) = \int_R p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathcal{Z}_{k-1})\,\mathrm{d}x_{k-1}$$

Channel Filters

Fusion With Other Estimates

$$p(\mathbf{x}_k|\mathcal{Z}_k^i) = p(\mathbf{x}_k|\mathbf{z}_k^{(i)}, \mathcal{Z}_{k-1}^i) \prod_{j\in\mathcal{N}_i} \frac{p(\mathbf{x}_k|\mathbf{z}_k^{(j)}, \mathcal{Z}_{k-1}^j)}{p(\mathbf{x}_k|\mathcal{Z}_{k-1}^i \cap \mathcal{Z}_{k-1}^j)}$$

$$\text{after fusion at } i = \text{before fusion at } i \prod_{\text{neighbours of } i} \frac{\text{before fusion at } j}{\text{before fusion at } ij}$$

- remark that

$$(\mathcal{Z}_{k-1}^i \cup \mathbf{z}_k^{(i)}) \cap (\mathcal{Z}_{k-1}^j \cup \mathbf{z}_k^{(j)}) = \mathcal{Z}_{k-1}^i \cap \mathcal{Z}_{k-1}^j$$

Channel Filters

Channel Filter ij

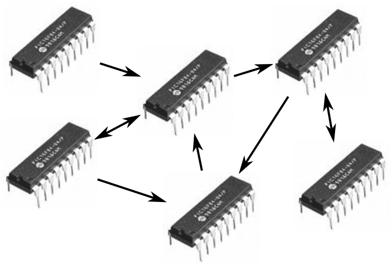
$$p(\mathbf{x}_k|\mathcal{Z}_k^i \cap \mathcal{Z}_k^j) = \frac{p(\mathbf{x}_k|\mathbf{z}_k^{(i)}, \mathcal{Z}_{k-1}^i)p(\mathbf{x}_k|\mathbf{z}_k^{(j)}, \mathcal{Z}_{k-1}^j)}{p(\mathbf{x}_k|\mathcal{Z}_{k-1}^i \cap \mathcal{Z}_{k-1}^j)}$$

$$\text{after communication at } ij = \\ = \frac{\text{before communication at } i \cdot \text{before communication at } j}{\text{before communication at } ij}$$

remember that

- $\mathcal{Z}_k^i \cap \mathcal{Z}_k^j = (\mathcal{Z}_{k-1}^i \cup \mathbf{z}_k^{(i)}) \cup (\mathcal{Z}_{k-1}^j \cup \mathbf{z}_k^{(j)})$
- only $p(\mathbf{x}_k|\mathbf{z}_k^{(\bullet)}, \mathcal{Z}_{k-1}^{\bullet})$ communicated
- need to add $|\mathcal{N}_i|$ channel filters to i-th estimator

General Decentralisation



- ad hoc structure
- time varying structure
- no global knowledge
- ...

Covariance Consistent Estimate

Covariance Consistence

estimate $\{\hat{\mathbf{x}}, \mathbf{P}\}$ is covariance consistent IF

$$\mathbf{P} - \mathrm{E}[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T] \underset{\text{positive semidefinite}}{\geq} 0$$

i.e. IF estimated error covariance is NOT underestimated

Convex Combination

combination of covariance consistent estimates $\{\hat{\mathbf{x}}_1, \mathbf{P}_1\}$, $\{\hat{\mathbf{x}}_2, \mathbf{P}_2\}$

$$\begin{aligned} \mathbf{P}^{-1}\hat{\mathbf{x}} &= \omega \mathbf{P}_1^{-1}\hat{\mathbf{x}}_1 + (1-\omega)\mathbf{P}_2^{-1}\hat{\mathbf{x}}_2, \\ \mathbf{P}^{-1} &= \omega \mathbf{P}_1^{-1} + (1-\omega)\mathbf{P}_2^{-1}, \end{aligned}$$

$\omega \in [0, 1]$, leads to covariance consistence of estimate $\{\hat{\mathbf{x}}, \mathbf{P}\}$
for UNKNOWN cross-covariance $\mathbf{P}_{12} = \mathrm{E}[(\mathbf{x} - \hat{\mathbf{x}}_1)(\mathbf{x} - \hat{\mathbf{x}}_2)^T]$

Covariance Intersection

Covariance Intersection Fusion Criterion

choose ω^* as

$$\arg\min_{\omega\in[0,1]}(\det \mathbf{P}) \quad \text{OR} \quad \arg\min_{\omega\in[0,1]}(\operatorname{tr} \mathbf{P})$$

Graphical Interpretation in 2D

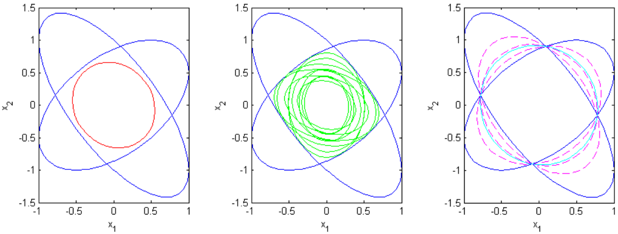


Figure: a)independent and b)dependent estimates fusion and c)CI fusion

Most Informative Estimate

Shannon Fusion

combine pdf's, $\omega \in [0, 1]$,

$$p_\omega(x) = \frac{p_1^\omega(x) p_2^{1-\omega}(x)}{\int_R p_1^\omega(x) p_2^{1-\omega}(x)\,\mathrm{d}x}$$

to minimize the entropy of the fused pdf (= Shannon information)

$$\mathcal{H}(p_\omega) = -\int_R p_\omega(x) \ln p_\omega(x)\,\mathrm{d}x$$

Minimal Probability of Error

Chernoff Fusion

combine pdf's, $\omega \in [0, 1]$,

$$p_\omega(x) = \frac{p_1^\omega(x) p_2^{1-\omega}(x)}{\int\limits_R p_1^\omega(x) p_2^{1-\omega}(x) \,\mathrm{d}x}$$

to minimize Chernoff information

$$\mathcal{C}(p_1, p_2) = - \min_{0 \le \omega \le 1} \left(\ln \int\limits_R p_1^\omega p_2^{1-\omega}(x) \,\mathrm{d}x \right)$$

Minimal Probability of Error

Chernoff Fusion Solution

the optimal ω^* satisfies

$$\mathcal{D}^* = \mathcal{D}(p_{\omega^*} \parallel p_1) = \mathcal{D}(p_{\omega^*} \parallel p_2)$$

where $\mathcal{D}(p_1 \parallel p_2)$ is Kullback-Leibler divergence

$$\mathcal{D}(p_1 \parallel p_2) = \int_R p_1(x) \ln\left(\frac{p_1(x)}{p_2(x)}\right) \mathrm{d}x$$

Summary

information fusion can be done via

- summing independent information
- computing dependcies
- conservative estimation