Bayesian modelling of train doors reliability

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Early work with Raffaele Argiento (CNR IMATI) and Enrico Cagno, Franco Caron, Mauro Mancini and Roberto Spreafico (Politecnico di Milano)

OUTLINE

- Statement of the problem
- Repairable systems and NHPPs
- *Part I*
  - All failure causes together
  - Univariate intensity function (kms as function of time)
- *Part II*
  - Failures divided according to causes
  - Bivariate intensity function (kms and time)
- Future research

STATEMENT OF THE PROBLEM

- New underground line opened in a major Italian city on 3/5/1990 just before the FIFA World Cup

- 40 trains delivered to the company between 11/1989 and 3/1991 and put on service between 4/1990 and 7/1992

- Interest in failures of some components:
  - doors (major cause of failures)
  - engine wheels
  - two converters

- Doors failure data collected between 1/4/1990 and 31/12/1998

- Data collected by B.Sc. student at Politecnico di Milano for his dissertation

STATEMENT OF THE PROBLEM

- Interest in process modelling and estimation
  - all doors failure regardless of the cause
  - only individual, major failure causes
- Interest in reliability check before warranty expiration
  - reliability standards set by the contract between manufacturer and transportation company
  - structural failures occurred during the warranty time are responsibility of the manufacturer (and the company's after warranty expiration)
  - transportation company can ask for manufacturer's intervention on trains if there is evidence of poor reliability during warranty time (but not later)

REPAIRABLE SYSTEMS

Theorem (due to Grigelionis, see Thompson, 1988, p.69) states that, *under suitable conditions*, superposition of *many* failure processes, one for each failure mode, is *approximately* a Poisson process

- Doors are complex systems made of many components, subject to different failure causes

- Upon failures, repairs are immediate (i.e. done in a negligible time w.r.t. doors lifetime) and minimal (i.e. just the cause of the failure is fixed)

- Repairs bring reliability back to its status just before failures (*bad as old* property)

$\Rightarrow$ Non-homogeneous Poisson process (NHPP)

NONHOMOGENEOUS POISSON PROCESS

- $N_t, t \geq 0$ # events by time t
- $N(y,s)$ # events in $(y,s]$
- $\Lambda(t) = \mathcal{E} N_t$ mean value function
- $\Lambda(y,s) = \Lambda(s) - \Lambda(y)$ expected # events in $(y,s]$

$N_t, t \geq 0$, NHPP with intensity function $\lambda(t)$ *iff*

1. $N_0 = 0$
2. independent increments
3. $\mathcal{P}\{\#\text{ events in }(t,t+h) \geq 2\} = o(h)$
4. $\mathcal{P}\{\#\text{ events in }(t,t+h) = 1\} = \lambda(t)h + o(h)$

$$\Rightarrow \mathcal{P}\{N(y,s) = k\} = \frac{\Lambda(y,s)^k}{k!} e^{-\Lambda(y,s)}, \forall k \in \mathcal{N}$$

NONHOMOGENEOUS POISSON PROCESS

$\lambda(t) \equiv \lambda \; \forall t \Rightarrow$ HPP

- $\lambda(t)$: intensity function of N_t
- $\lambda(t) := \lim_{\Delta \to 0} \frac{\mathcal{P}\{N(t, t+\Delta] \geq 1\}}{\Delta}, \; \forall t \geq 0$
- $\mu(t) := \frac{d\Lambda(t)}{dt}$: Rocof (rate of occurrence of failures)

Property 3. $\Rightarrow \mu(t) = \lambda(t)$ a.e. $\Rightarrow \Lambda(y, s) = \int_y^s \lambda(t)dt$

$\lambda(t; \theta) \Rightarrow$ inference on θ

- Failures $\underline{T} = (t_1, \ldots, t_n)$ in $(0, y]$
- $\Rightarrow$ likelihood $L(\theta \mid \underline{T}) = \prod_{i=1}^{n} \lambda(t_i; \theta) e^{-\Lambda(y;\theta)}$

FAILURE MODELS

- All door failures together regardless of cause or separated by cause
- A NHPP used directly to model failures or ...
- ... NHPP for each *independent* cause and apply

  Superposition Theorem: Consider n independent Poisson processes $N_t^{(i)}, t \geq 0$, with intensity function $\lambda^{(i)}(t)$, $i = 1, \ldots, n$. The sum process $N_t, t \geq 0$, defined as $N_t = \sum_{i=1}^n N_t^{(i)}$ is a Poisson process with intensity function $\lambda(t) = \sum_{i=1}^n \lambda^{(i)}(t)$

- *In any case*, a NHPP for the failures

FAILURE MODELS

- Trains could be considered as **a**) *different* systems or **b)** the *same, unique* system
  - **a**) likelihood as product of likelihoods based upon individual intensities
  - **b)** likelihood based upon sum of intensities (as a consequence of the Superposition Theorem)

- Behaviour of different trains could be ...
  - **i)** *equal* $\Rightarrow$ same parameters
  - **ii)** *similar* $\Rightarrow$ different parameters from same distribution (*exchangeability*)
  - **iii)** *different* $\Rightarrow$ different parameters from different distributions
  - **iv)** *"almost" different* $\Rightarrow$ some common parameters and other different ones from different distributions

- EDA lead to **iii)** in Part I, and further physical considerations lead to **iv)** in Part II

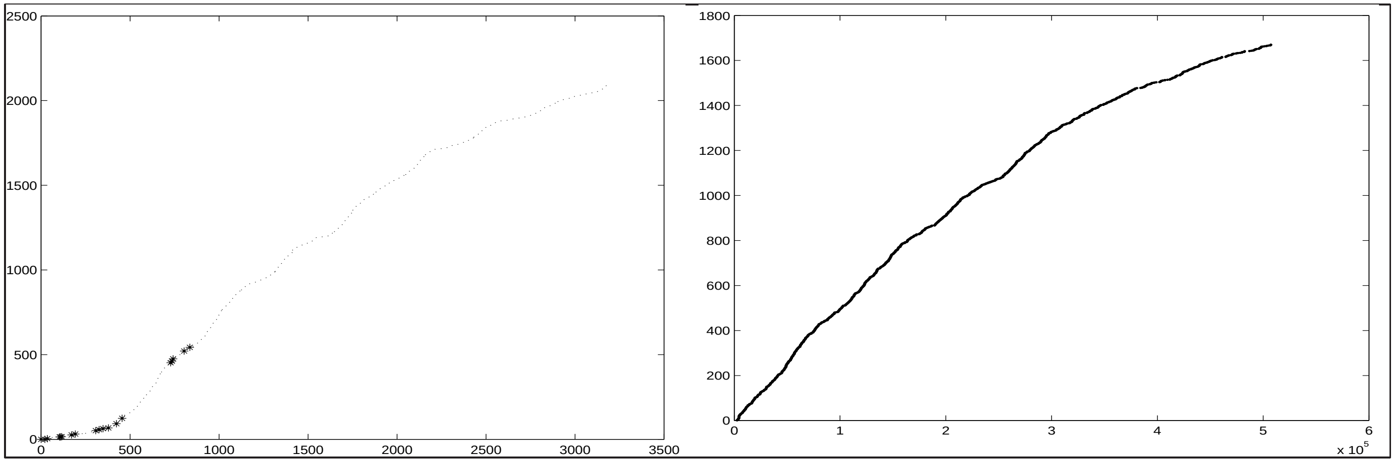
FAILURE MODELS

- Likelihood based on failures in one train
  - Estimation of its intensity function
  - Comparison between its cumulative and estimated expectation of number of failures
  - Predictive distribution and expectation of its future failures
- Likelihood based on failures in all train
  - Comparison between cumulative and estimated expectation of number of failures among all trains
  - Predictive distribution and expectation of future failures of a new train

DOUBLE SCALE DATA

Data: more than 2000 door failures of 40 trains, put on service from 1/4/1990 to 20/7/1992, observed up to 31/12/1998

Goal: checking components reliability before warranty's expiration



Failures vs. days (left) and failures vs. kilometers (right)

- Concavity denotes improvement over time
- Oscillations
- Transient behaviour during first 500 days

DOUBLE SCALE DATA

Nelson (1995)

- n blood analysers processing s_i specimen in t_i hours

- Independent failures $Y_i \sim \mathcal{P}(\nu s_i + \lambda t_i)$, $i = 1, n$

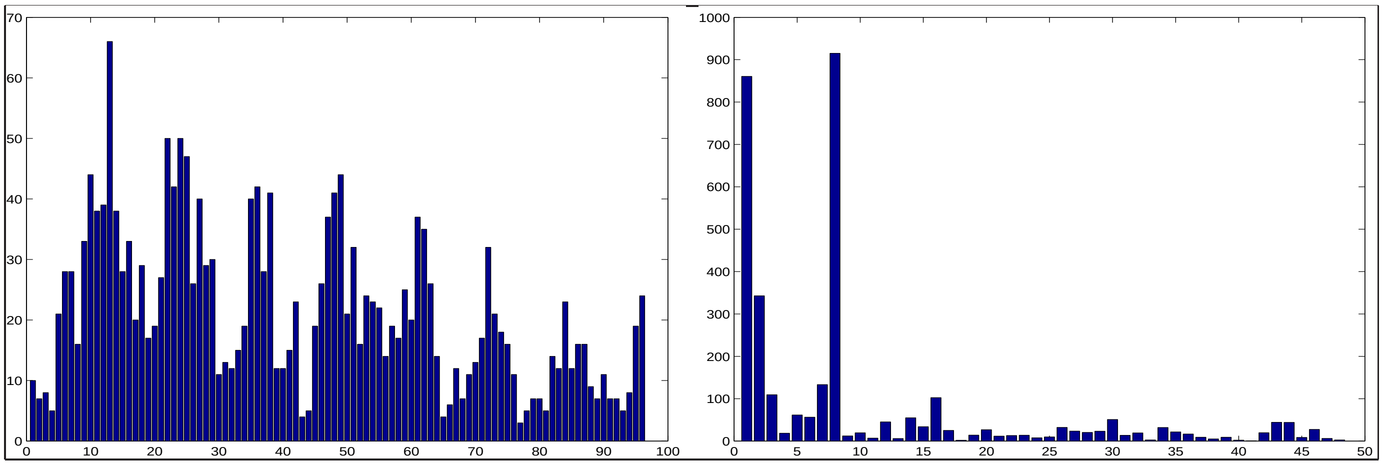
- MLE and C.I's of ν and λ

DOUBLE SCALE DATA

Singpurwalla and Wilson (1998) - Additive hazards model

- T failure time
- $M(t)$ usage at time t
- $U =_d M(T)$ usage at failure time T
- Interest in $f_{TU}(t,u) = f_{T|M(t)}(t|u) f_{M(t)}(u)$
- Additive hazards function of T: $\lambda(t) = \lambda_0(t) + \eta M(t)$
- $M(t)$ compound or doubly stochastic Poisson, Gamma process

SEASONALITY



Left: Monthly no. of failures for the 40 trains starting January 1991

Right: Spectrum of the time series of the monthly number of failures from 1991 to 1998

- Decreasing trend
- Periodicity (estimated at 12 months by the spectrum)
- NHPP: $\lambda(t) = \exp\{\alpha + \rho \sin(\omega t + \theta)\}$ (Lewis, 1970, 1972; Vere-Jones and Ozaki, 1982)

FULLY PARAMETRIC MODEL

Marked Poisson process on time scale

$\lambda(t; \theta_1, \theta_2) = \mu(g(t); \theta_1) s(t; \theta_2)$

- $\mu(k; \theta_1) = \beta_0 \dfrac{\log(1 + \beta_1 k)}{(1 + \beta_1 k)}$
  - $\mu(0; \theta_1) = 0$, maximum at $(e - 1)/b_1$ and $\lim_{k \to \infty} u(k; \theta_1) = 0$
  - m.v.f. $\Lambda(k) = \beta_0 \log^2(1 + \beta_1 k)/(2\beta_1)$
    suitable for actual cumulative number of failures
- $s(t; \theta_2) = \exp\{\rho \cos(\omega t + \varphi)\}$(periodic component)
- kilometers $k|t \sim \mathcal{N}(g(t), \sigma^2)$
- $E(k|t) = g(t) = at + bt^2$

FULLY PARAMETRIC MODEL

- j-th train monitored in $[0, T_j]$
- Failures at times $(t_1, \dots, t_{n_j}) = \mathbf{t_j}$ and kilometers $(k_1, \dots, k_{n_j}) = \mathbf{k_j}$
- Likelihood for j-th train

$$L_j(\theta_1, \theta_2) = \prod_{i=1}^{n_j} \mu(g(t_i); \theta_1) s(t_i; \theta_2) \exp\left[- \int_0^{T_j} \mu(g(t); \theta_1) s(t; \theta_2) dt\right]$$

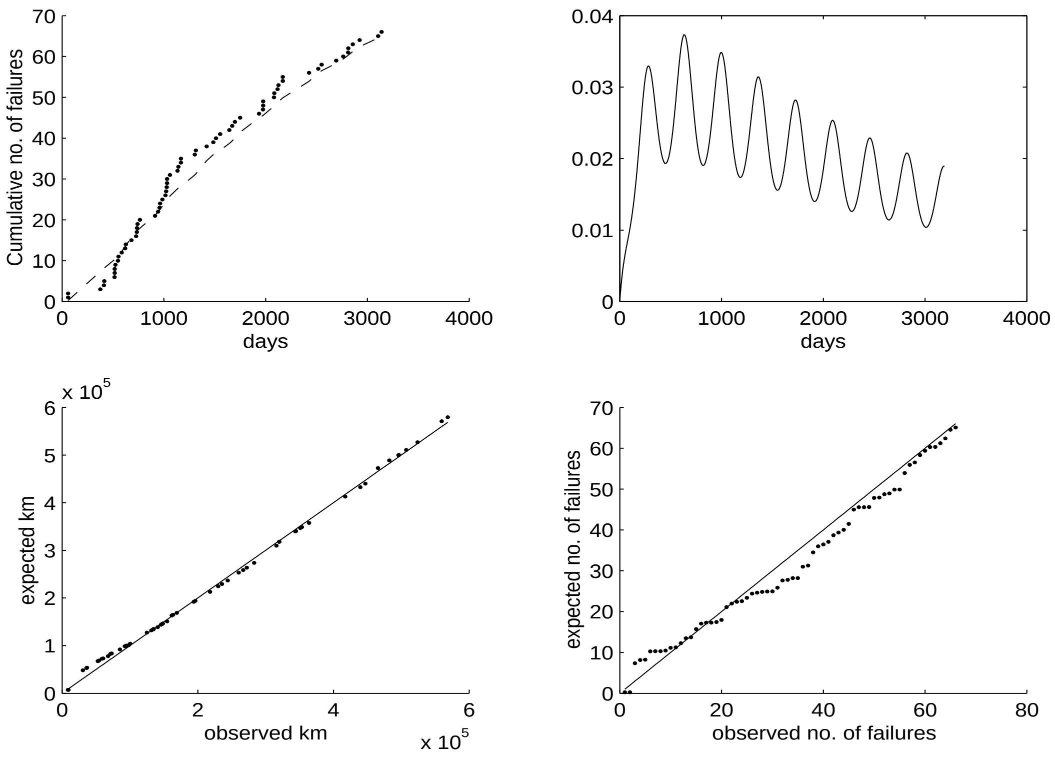
Parameter	MLE	C.I.	Parameter	MLE	C.I.
$a \times 10^{-2}$	1.209	[1.171, 1.247]	$b \times 10^{2}$	2.025	[1.862, 2.188]
$\sigma^2 \times 10^{-7}$	5.809	[4.214, 8.345]	$\rho \times 10$	3.234	[0.000, 6.779]
$\beta_0 \times 10^{2}$	7.358	[5.640, 9.076]	$\beta_1 \times 10^{5}$	2.239	[1.938, 2.540]

DIAGNOSTIC FOR ONE TRAIN

Theorem 1 *Let* $\Lambda(t)$ *be a continuous nondecreasing function. Then* $T_1, T_2, \ldots$ *are arrival times in a Poisson process* N_t *with m.v.f.* $\Lambda(t)$ *if and only if* $\Lambda(T_1), \Lambda(T_2), \ldots$ *are arrival times in an HPP* H_t *with failure rate one.*

- $\widehat{\Lambda}(t)$ estimated from data $T_1, T_2, \ldots$
- *Suppose* $T_1, T_2, \ldots$ from NHPP with m.v.f. $\widehat{\Lambda}(t)$
- $Y_1 = \widehat{\Lambda}(T_1), Y_2 = \widehat{\Lambda}(T_2), \ldots$ *data* from HPP with rate 1
- Interarrival times $X_i = Y_i - Y_{i-1}$ i.i.d. $\mathcal{E}(1)$
- $U_i = \exp\{-X_i\}$ i.i.d. $\mathcal{U}[0, 1]$
- Should $\widehat{\Lambda}(t)$ be the right model, then U_i's should be uniform r.v.'s
- Kolmogorov-Smirnov test about data from uniform distribution
- **Unsatisfactory results**

DIAGNOSTIC PLOTS FOR ONE TRAIN



Estimated m.v.f. vs. observed failures (top left), estimated intensity function (top right), expected vs. observed odometer readings at failure times (bottom left) and expected vs. observed number of failures (bottom right)

HIERARCHICAL MODEL

- Hierarchical model with $g(t)$ realisation of a Gamma process

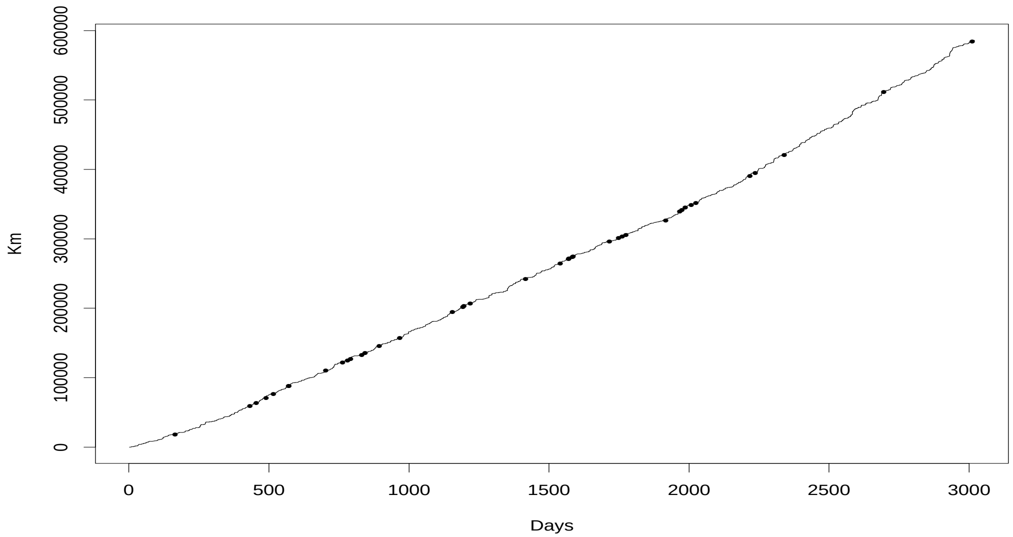
$$
\begin{aligned}
g(t) &\sim \mathcal{G}(at, b) \\
\theta &\sim \pi(\theta) \\
[\mathbf{t} \mid \mathbf{g}, \theta] &= NHPP\{\mu(g(t); \theta_1) s(t; \theta_2)\} \\
[\mathbf{k} \mid \mathbf{t}, \mathbf{g}] &= \prod_{i=1}^{n} \delta_{g(t_i)}(\cdot)
\end{aligned}
$$

- MCMC algorithm (Gibbs sampling with Metropolis steps within)

GENERATION OF g

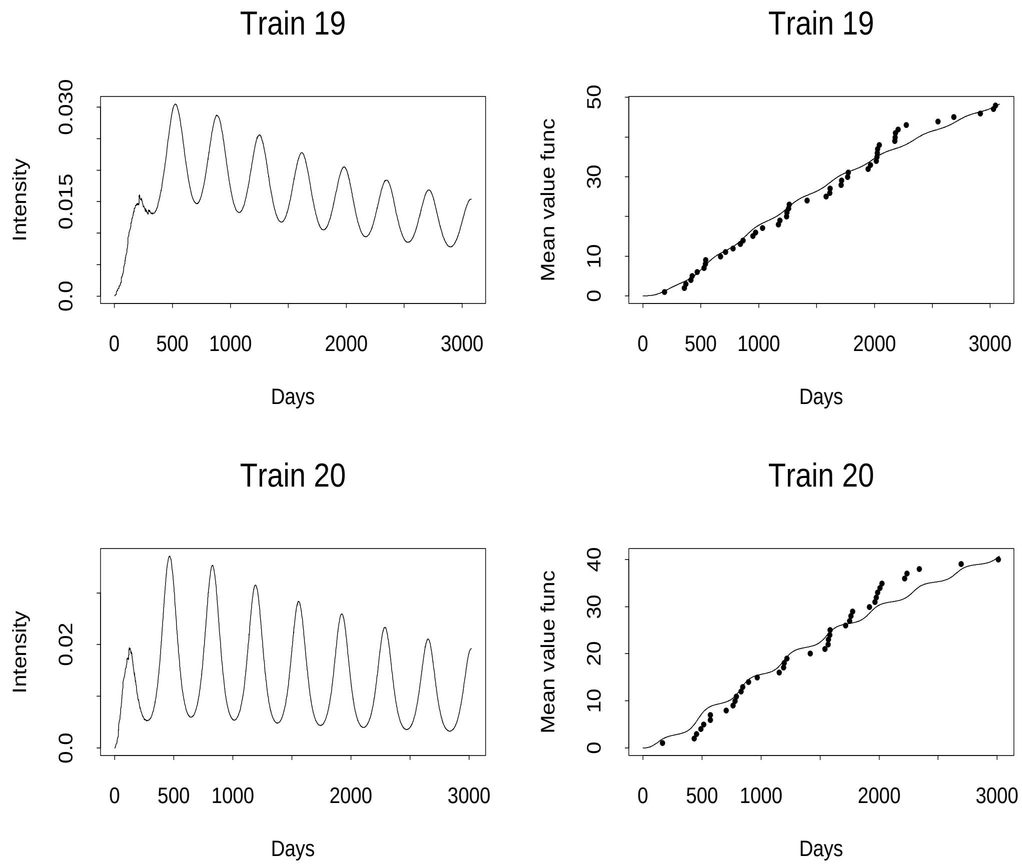
- g updated with an acceptance/rejection step
- g needs to go through observed failure data $k_i = g(t_i)$
- link between Dirichlet and Gamma processes
- $g(t)$ points drawn from the cumulative distribution of a Dirichlet process, multiplied by $g(t_i) - g(t_{i-1})$ and shifted above by $g(t_{i-1})$

GENERATION OF g

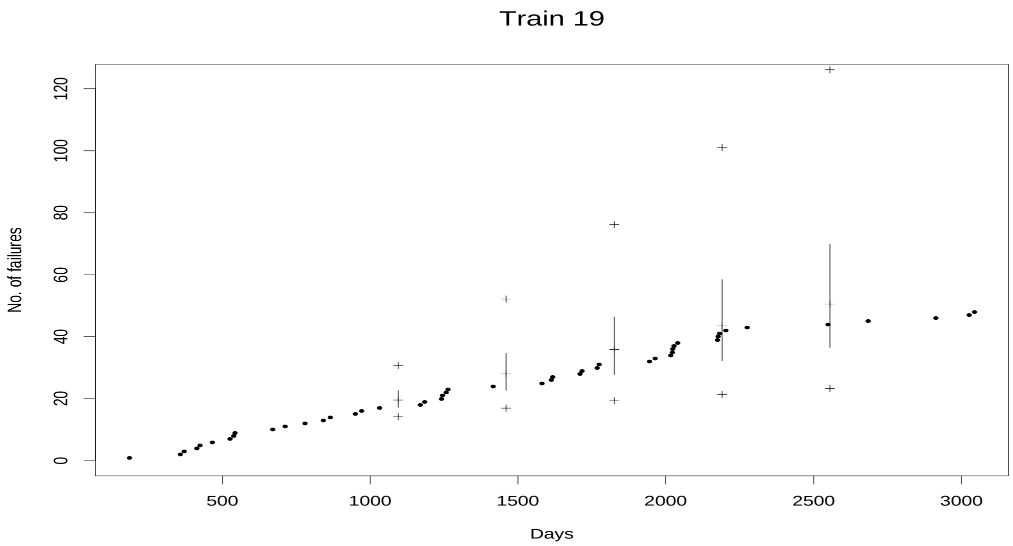


An example of g during the MCMC run

INTENSITY AND MEAN VALUE FUNCTION ESTIMATION



FORECAST



Prediction intervals of the number of failures for train 19 using 730 days (2 years) of observations, up to 5 years ahead. The vertical lines are the interquartile intervals with the posterior median; the plus signs are the extremes of 95% posterior probability intervals

DIFFERENT FAILURE MODES MODEL

- Identification of most relevant failure modes

- Models for relevant failure modes

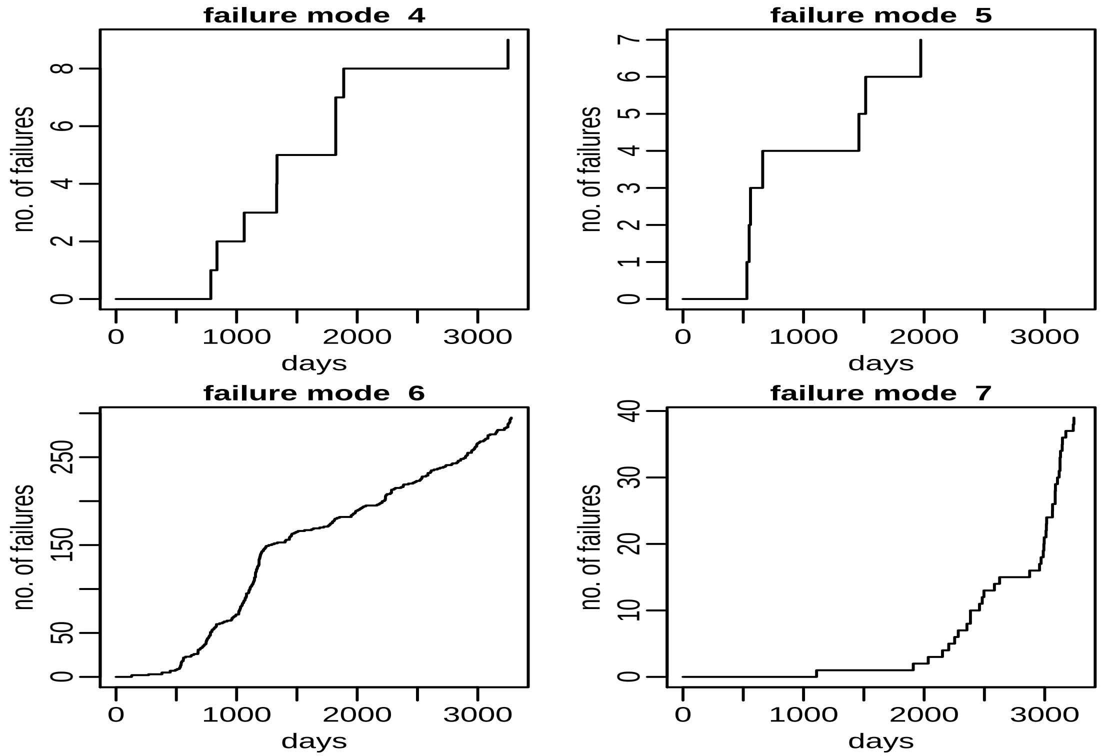
- Trains treated individually but combined later

DIFFERENT FAILURE MODES MODEL

Code	Subsystem	No. of parts	Total failures
1	opening commands (electrical)	14	530
2	cables and clamps	4	33
3	mechanical parts	67	1182
4	electrical protections	12	9
5	power supply circuit	2	7
6	pneumatic gear	31	295
7	electro-valves	8	39

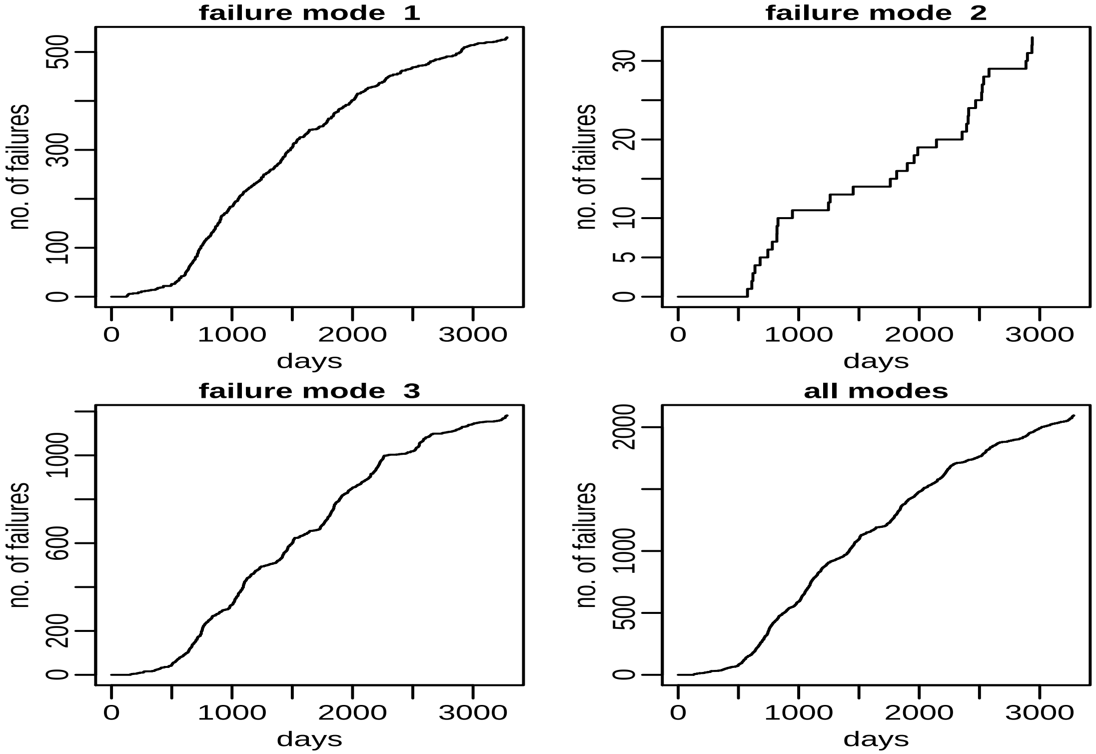
Classification of failure modes and total failures per mode for all trains in nine years

CUMULATIVE NUMBER OF FAILURES



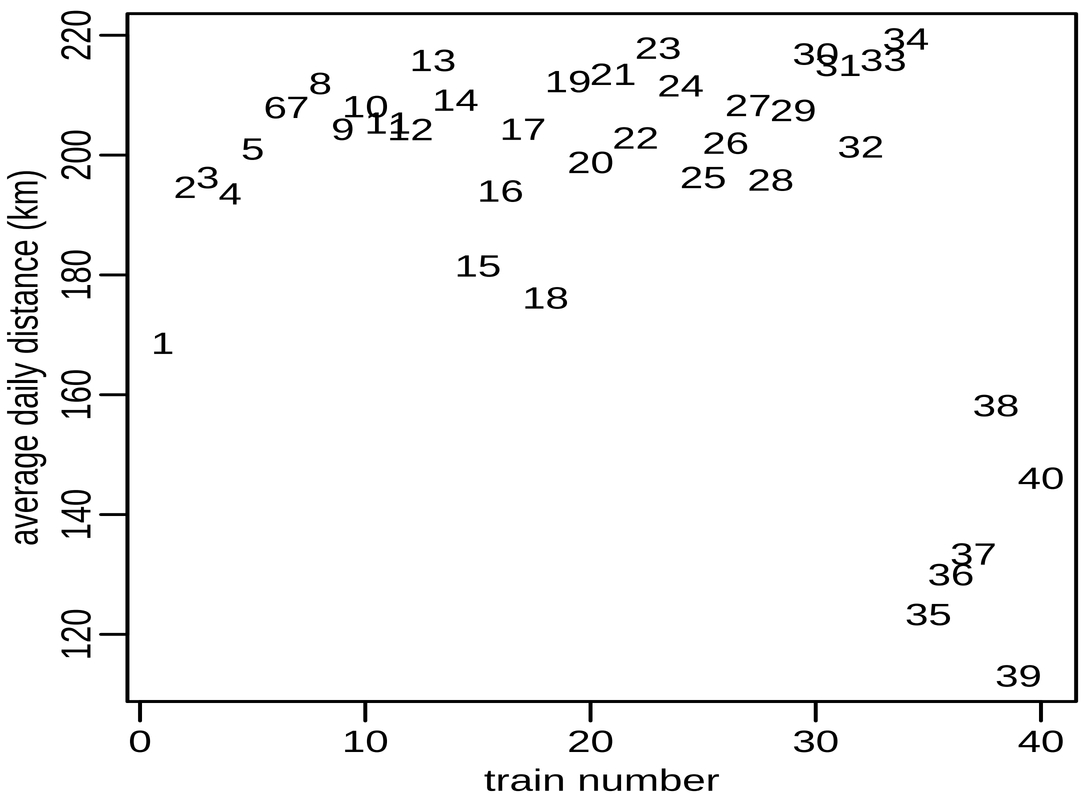
- Failure modes 4 and 5 very rare $\Rightarrow$ not enough information for fitting a stochastic process model
- Failure modes 6 and 7 show change-points (F.R. and Sivaganesan, 2005)

CUMULATIVE NUMBER OF FAILURES



- Failure modes 1, 2 and 3 display a more regular pattern
- Mode 2 failures are only 0.11 per train and per year
- $\Rightarrow$ concentrate on failure modes 1 and 3

AVERAGE DAILY DISTANCE



- Different average daily distance
- More recent trains are used less daily

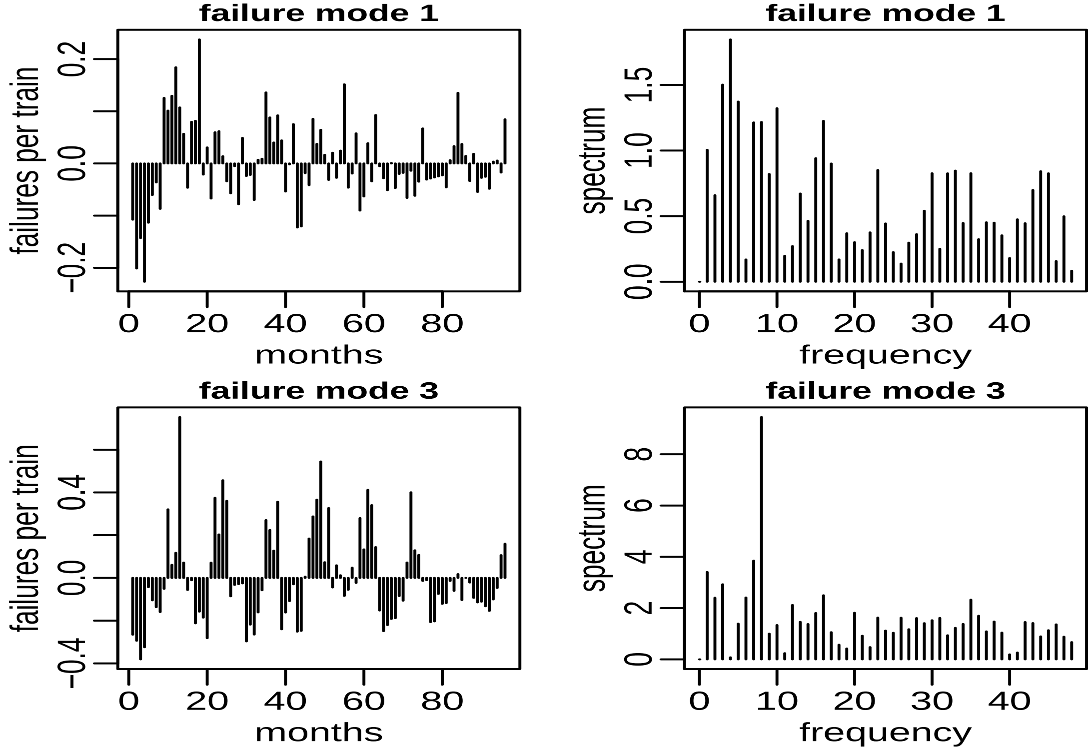
BIVARIATE INTENSITY FUNCTION

For each train i

$$\begin{aligned}\lambda_i(t,s) &= \mu \exp\left\{-\gamma(s-a_i-c_i(t-t_{0i}))^2 w(t-t_{0i})\right\} \cdot \\ &\quad \cdot \exp\left\{A\cos(\omega(t-d))\right\} \lambda_0(t-t_{0i})\end{aligned}$$

- t_{0i} starting operation date
- $a_i + c_i(t-t_{0i})$ expected distance after $(t-t_{0i})$ days in service ((a_i, c_i) different for every train, as seen before)
- $w(\cdot)$ positive weight function, rather close to 0 for $(t-t_{0i}) \approx 0$ and to 1 for $(t-t_{0i})$ large (initial relation between distance and time not linear)
  e.g. $w(z) = \dfrac{\sqrt{1+z}}{1+\sqrt{1+z}}$, bounded between 0.5 and 1
- $\lambda_0(\cdot)$ is a baseline intensity function (depending on time since first ride), common to all trains except for starting point
- exponentiated cosine is a periodic component with phase d (depending on calendar time), common to all trains

PERIODIC COMPONENT



- Periodogram of monthly time series of failure modes 1 and 3 (after detrending)
- No clear frequency for failure mode 1 $\Rightarrow$ omit periodic component in intensity
- 12-month cycle evident for failure mode 3

BASELINE INTENSITY

Choice of baseline function $\lambda_0(t)$ with m.v.f.

- $\Lambda_0(u) = Mu^b$ (Power Law process)
- $\Lambda_0(u) = \ln(1 + bu)$ (Reciprocal)
- $\Lambda_0(u) = (1 - e^{-bt})/b$ (Exponential)

Many other choices are possible ...

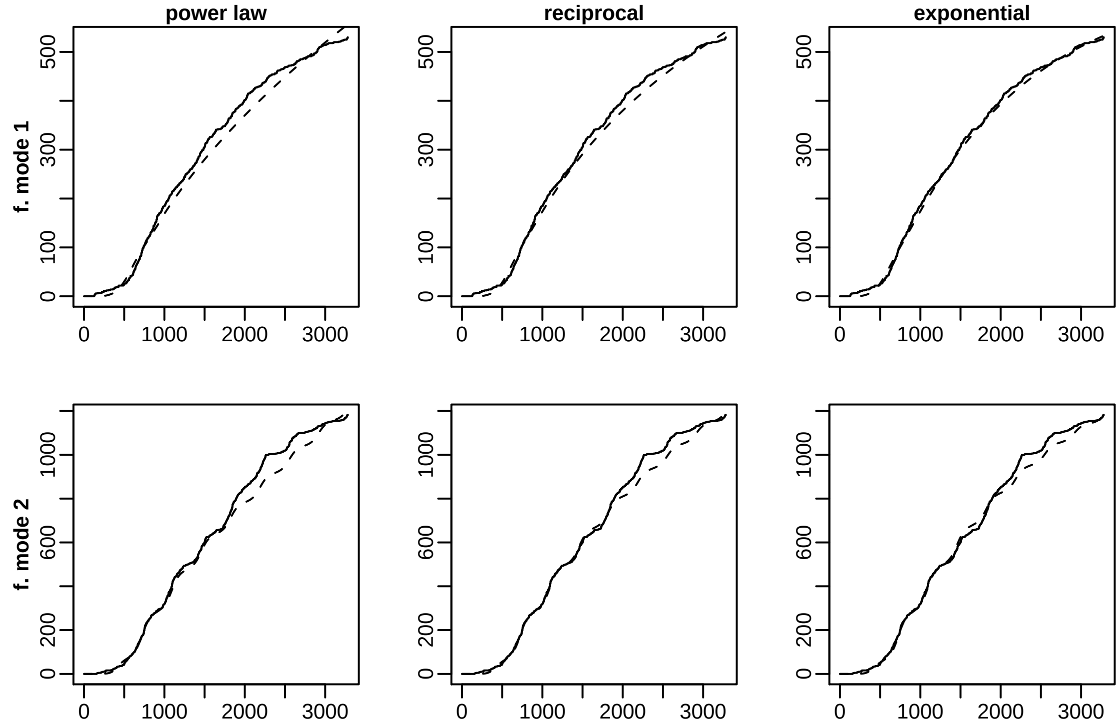
We omit writing likelihood, priors, posterior conditionals and MCMC implementation

ESTIMATE OF MEAN VALUE FUNCTION

- Posterior mean of $\Lambda(t;\theta)$
  - correct one
  - requires numerical integration of $\lambda(t;\theta)$ at each MCMC step

- Plot of $\Lambda(t;\widehat{\theta}) = \sum_{i=1}^{40} \int_{t_{0i}}^{t} \lambda_i(u;\widehat{\theta})\, du, \qquad t = 1,\ldots,3287$
  - $\widehat{\theta}$ estimate of θ from MCMC run
  - $\lambda_i(t) = \mu \sqrt{\frac{\pi}{\gamma w(t-t_{0i})}}\, \Phi\left\{(a_i + c_i(t-t_{0i}))\sqrt{2\gamma w(t-t_{0i})}\right\} \cdot$
    $\cdot \exp\{A \cos(\omega\,(t-d))\}\ \lambda_0(t-t_{0i})$
    (marginal of $\lambda_i(t,s)$)
  - not optimal but useful

ESTIMATE OF MEAN VALUE FUNCTION



- Cumulative number of failures for all trains and estimated mean value function (dashed)
- Row 1: failure mode 1; Row 2: failure mode 3
- Each column is for a different baseline (exponential in third column is the best)

FORECAST OF FUTURE FAILURES OF GIVEN MODE

- D_{T_0} data available at day T_0
- $\pi(\cdot \mid D_{T_0})$ posterior density of θ

Predictive distribution

$$P(N_{T_0+u} - N_{T_0} = x \mid D_{T_0}) = \int e^{-\{\Lambda(T_0+u;\theta)-\Lambda(T_0;\theta)\}} \frac{\{\Lambda(T_0+u;\theta)-\Lambda(T_0;\theta)\}^x}{x!} \pi(\theta \mid D_{T_0}) \, d\theta$$

Expected value

$$\mathsf{E}(N_{T_0+u} - N_{T_0} \mid D_{T_0}) = \int \{\Lambda(T_0+u;\theta) - \Lambda(T_0;\theta)\} \, \pi(\theta \mid D_{T_0}) \, d\theta$$

FORECAST OF FUTURE FAILURES OF MODE 1

end of recording period	forecasting horizon (years)	95% credibility interval	true value	posterior mean
1992	1	(86, 143)	83	114
	2	(79, 140)	72	109
	3	(71, 138)	62	105
1993	1	(69, 124)	72	97
	2	(59, 121)	62	90
	3	(50, 119)	42	85
1994	1	(50, 100)	62	74
	2	(41, 95)	42	66
	3	(32, 91)	35	59
1995	1	(38, 81)	42	59
	2	(30, 74)	35	51
	3	(24, 68)	23	44
1996	1	(27, 60)	35	43
	2	(20, 52)	23	35
1997	1	(19, 46)	23	39

FORECAST OF FUTURE FAILURES OF MODE 3

end of recording period	forecasting horizon (years)	95% credibility interval	true value	posterior mean
1992	1	(159, 255)	165	209
	2	(137, 255)	169	196
	3	(117, 261)	188	184
1993	1	(132, 213)	169	171
	2	(111, 205)	188	154
	3	(92, 198)	124	139
1994	1	(120, 189)	188	153
	2	(102, 177)	124	138
	3	(87, 167)	74	124
1995	1	(132, 200)	124	165
	2	(120, 193)	74	156
	3	(109, 188)	52	147
1996	1	(105, 164)	74	134
	2	(93, 153)	52	122
1997	1	(71, 117)	52	94

FAILURE FORECAST OF NEW TRAIN

- $N_H(t)$ failure counting process for new train
- $\lambda_H(t;\theta)$ its intensity function
- D_t failure data up to time t
- T_0 two years

$$\Pr(N_H(T_0) > x_U \mid D_t) = 1 - \int \sum_{x=0}^{x_U} e^{-\Lambda_H(T_0;\theta)} \frac{[\Lambda_H(T_0;\theta)]^x}{x!} \pi(\theta \mid D_t) \, d\theta$$

f. mode 1	x_U	3	4	5	6	7	8	9	10	11	12	13
	prob.	0.82	0.68	0.52	0.36	0.23	0.14	0.07	0.04	0.02	0.01	0.00
f. mode 3	x_U	12	13	14	15	16	17	18	19	20	21	22
	prob.	0.47	0.36	0.26	0.18	0.12	0.08	0.05	0.03	0.01	0.01	0.00

FUTURE RESEARCH

- Exchangeable parameters
- Bayesian nonparametrics for the baseline intensity
- Different repair situations (e.g. imperfect repair)
- Optimal maintenance policy (to prevent failures)

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