A Statistical Approach to Local Evaluation of a Single Texture Image

Jiří Grim & Petr Somol

Institute of Information Theory and Automation
Academy of Sciences of the Czech Republic, Prague

Department of Pattern Recognition
http://www.utia.cas.cz/RO

Seminar DAR, UTIA Prague, April 2007

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Local Statistical Texture Model

Assumption:

homogeneous texture, i.e. local statistical properties of pixels in a suitably chosen search window are shift-invariant

Digitized grey-scale texture: $\mathcal{Y} = [y_{ij}]_{i=1}^{I}{}_{j=1}^{J}, \quad y_{ij} \approx$ grey-levels

window patch: $\mathbf{x} = (x_1, x_2, \ldots, x_N) \in \mathcal{X}, \quad \mathcal{X} = R^N$

Method:

approximation of the joint multivariate probability density of window pixels by normal mixture of product components

$$P(\mathbf{x}) = \sum_{m \in \mathcal{M}} w_m F(\mathbf{x}|\boldsymbol{\mu}_m, \boldsymbol{\sigma}_m) = \sum_{m \in \mathcal{M}} w_m \prod_{n \in \mathcal{N}} f_n(x_n|\mu_{mn}, \sigma_{mn})$$

$$f_n(x_n|\mu_{mn}, \sigma_{mn}) = \frac{1}{\sqrt{2\pi}\sigma_{mn}} \exp\{-\frac{(x_n - \mu_{mn})^2}{2\sigma_{mn}^2}\}$$

$\mathcal{M} = \{1, \ldots, M\}, \quad \mathcal{N} = \{1, \ldots, N\} \approx$ index sets

EM Algorithm for Normal Mixtures

dat set: $\mathcal{S} = \{\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(K)}\}$ $\approx$ by scanning the image with the search window

$$F(\mathbf{x}|\boldsymbol{\mu}_m, \boldsymbol{\sigma}_m) = \prod_{n \in \mathcal{N}} \frac{1}{\sqrt{(2\pi)}\sigma_{mn}} \exp\left\{ -\frac{(x_n - \mu_{mn})^2}{2\sigma_{mn}^2} \right\}, \quad m \in \mathcal{M}$$

log-likelihood criterion:

$$L = \frac{1}{|\mathcal{S}|} \sum_{\mathbf{x} \in \mathcal{S}} \log P(\mathbf{x}) = \frac{1}{|\mathcal{S}|} \sum_{\mathbf{x} \in \mathcal{S}} \log[\sum_{m \in \mathcal{M}} w_m F(\mathbf{x}|\boldsymbol{\mu}_m, \boldsymbol{\sigma}_m)]$$

EM Algorithm:

$$q(m|\mathbf{x}) = \frac{w_m F(\mathbf{x}|\boldsymbol{\mu}_m, \boldsymbol{\sigma}_m)}{\sum_{j \in \mathcal{M}} w_j F(\mathbf{x}|\boldsymbol{\mu}_j, \boldsymbol{\sigma}_j)}$$

$$w_m^{'} = \frac{1}{|\mathcal{S}|} \sum_{\mathbf{x} \in \mathcal{S}} q(m|\mathbf{x}), \qquad \mu_{mn}^{'} = \frac{1}{\sum_{\mathbf{x} \in \mathcal{S}} q(m|\mathbf{x})} \sum_{\mathbf{x} \in \mathcal{S}} x_n q(m|\mathbf{x})$$

$$(\sigma_{mn}^{'})^2 = -(\mu_{mn}^{'})^2 + \frac{1}{\sum_{\mathbf{x} \in \mathcal{S}} q(m|\mathbf{x})} \sum_{\mathbf{x} \in \mathcal{S}} x_n^2 q(m|\mathbf{x})$$

Computational Details of Model Estimation

- high dimension of the window space $N \approx 10^2 - 10^3$
- low-frequency details $\Rightarrow$ large window-size $\Rightarrow$ high dimension
- dimension $N \approx 10^3$ becomes computationally prohibitive
- training data set $\mathcal{S}$ obtained by scanning the image with the search window (source texture image: 500x500 pixels $\Rightarrow$ $|\mathcal{S}| \approx 250000$)
- number of components $M \approx 10^1 - 10^2$
- EM algorithm: random initialization, 10 - 20 iterations
- (!) data vectors obtained by scanning the image with the search window are overlapping and therefore not independent
- data set $\mathcal{S}$ corresponds only to a "trajectory" in $\mathcal{X}$ produced by scanning the image ($\Rightarrow$ less representative, bad sampling properties)
- high-dimensional spaces are "sparse" $\Rightarrow$ nearly non-overlapping components, $q(m|\mathbf{x})$ behave nearly binary:

$$q_{\max}(\mathbf{x}) = \arg\max_m \{q(m|\mathbf{x})\}, \quad \bar{q}_{\max} = \frac{1}{|\mathcal{S}|} \sum_{\mathbf{x} \in \mathcal{S}} q_{\max}(\mathbf{x}) \approx 0.95 \div 0.99$$

Sequential Texture Synthesis by Local Prediction

$\mathcal{D} = \{j_1, \ldots, j_l\} \subset \mathcal{N}$ ≈ defined part of the window

$\mathcal{C} = \{i_1, \ldots, i_k\} = \mathcal{N} \setminus \mathcal{D}$ ≈ predicted part of the window

$$\mathbf{x}_D = (x_{j_1}, \ldots, x_{j_l}) \in \mathcal{X}_D, \quad F(\mathbf{x}_D|\boldsymbol{\mu}_m, \boldsymbol{\sigma}_m) = \prod_{n \in D} f_n(x_n|\mu_{mn}, \sigma_{mn})$$

$$\mathbf{x}_C = (x_{i_1}, \ldots, x_{i_k}) \in \mathcal{X}_C, \quad F(\mathbf{x}_C|\boldsymbol{\mu}_m, \boldsymbol{\sigma}_m) = \prod_{n \in C} f_n(x_n|\mu_{mn}, \sigma_{mn})$$

conditional distribution:

$$P_{C|D}(\mathbf{x}_C|\mathbf{x}_D) = \frac{P_{CD}(\mathbf{x}_C, \mathbf{x}_D)}{P_D(\mathbf{x}_D)} = \sum_{m \in \mathcal{M}} W_m(\mathbf{x}_D) F(\mathbf{x}_C|\boldsymbol{\mu}_{mC}, \boldsymbol{\sigma}_{mC})$$

$$W_m(\mathbf{x}_D) = \frac{f(m)F(\mathbf{x}_D|\boldsymbol{\mu}_{mD}, \boldsymbol{\sigma}_{mD})}{\sum_{j \in \mathcal{M}} f(j)F(\mathbf{x}_D|\boldsymbol{\mu}_{jD}, \boldsymbol{\sigma}_{jD})}$$

PREDICTION: expectation $\bar{\mathbf{x}}_C$ given $\mathbf{x}_D$

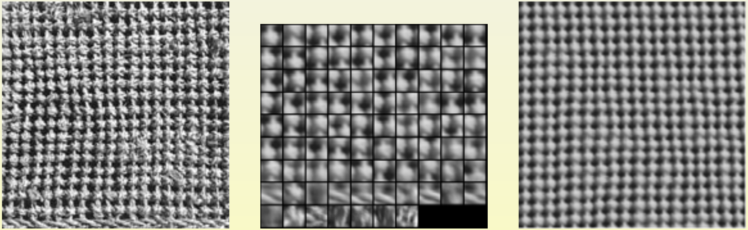
$$\bar{\mathbf{x}}_C = \mathrm{E}_{C|D}\{\mathbf{x}_C|\mathbf{x}_D\} = \int \mathbf{x}_C P_{C|D}(\mathbf{x}_C|\mathbf{x}_D) d\mathbf{x}_C = \sum_{m \in \mathcal{M}} W_m(\mathbf{x}_D)\boldsymbol{\mu}_{mC}$$

(alternatively: random sampling by $W_m(\mathbf{x}_D)$)

Example 1: Synthesis of the Texture "Ratan"

Image Synthesis: by random sampling the component means μ_{mC} according to the conditional weights $W_m(\mathbf{x}_D)$:

original image | component means | sampling of means

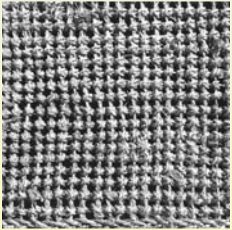


- source texture image: 512x512 pixels $\Rightarrow$ $|\mathcal{S}| \doteq 233000$
- dimension: $N = 30x30 = 900$, number of components: $|\mathcal{M}| = 80$
- number of EM iterations: $t = 15$

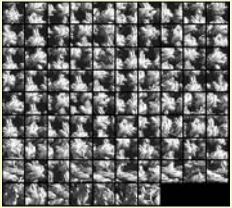
Example 1a: Synthesis of the Texture "Ratan"

"Realistic" Image Synthesis: component means μ_m replaced by similar pieces (patches) μ_m^* optimally chosen from the original texture:

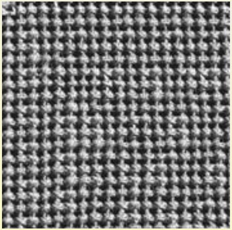
original image



texture patches



sampling of patches



$$\mu_m^* = \arg\min_{\mathbf{x}\in\mathcal{S}}\{\|\mathbf{x}-\mu_m\|^2\}.$$

▸ Other examples

Local Evaluation of the Log-Likelihood Values

Motivation:

successful texture synthesis $\Rightarrow$ the properties of source texture image can be described locally by the mixture model $P(\mathbf{x})$

LOG-LIKELIHOOD:

$\log P(\mathbf{x})$ $\approx$ typicality of window patch $\mathbf{x}$ $\longrightarrow$ grey-level at centr of $\mathbf{x}$

Remark: $\log P(\mathbf{x})$ is highly sensitive to grey-level deviation

LOG-LIKELIHOOD RATIO:

$\log P(\mathbf{x})/P_0(\mathbf{x})$ $\approx$ “structural” typicality of window patch $\mathbf{x}$

$$P_0(\mathbf{x}) = \prod_{n\in\mathcal{N}} f_n(x_n|\mu_{0n},\sigma_{0n}), \quad \mu_{0n} = \frac{1}{|\mathcal{S}|}\sum_{\mathbf{x}\in\mathcal{S}} x_n, \quad \sigma_{0n}^2 = \frac{1}{|\mathcal{S}|}\sum_{\mathbf{x}\in\mathcal{S}} x_n^2 - \mu_{0n}^2$$

Remark: μ_{0n}, σ_{0n} are nearly identical for all $n \in \mathcal{N}$

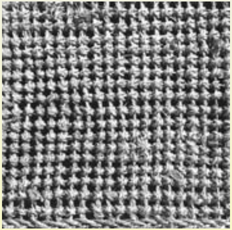
$\Rightarrow$ $\log P_0(\mathbf{x})$ is nearly invariant to pixel order

$\Rightarrow$ $\log P(\mathbf{x})/P_0(\mathbf{x})$ is nearly invariant with respect to grey-level deviations and it is more sensitive to structural irregularities.

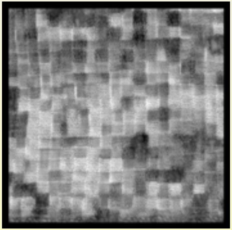
Example 1: Local Evaluation of Texture Image "Ratan"

Principle: log-likelihood values displayed as grey-levels at center pixel of the search window

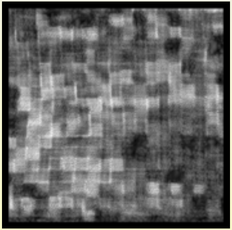
original image



log-likelihood



log-likelihood ratio

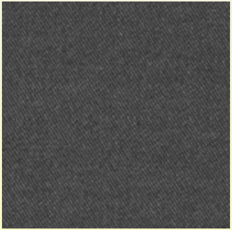


Remark: Log-likelihood ratio is less dependent on the grey-level mean and it is more sensitive to structural differences. The structural irregularities of the "ratan" texture (cf. left image) are therefore more clearly identified by the log-likelihood ratio (right image) than by the log-likelihood alone (central image)

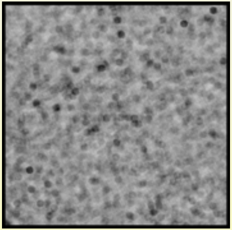
Example 2: Local Evaluation of Texture Image "Cushion"

Principle: log-likelihood values are displayed as grey-levels at center pixel of the search window

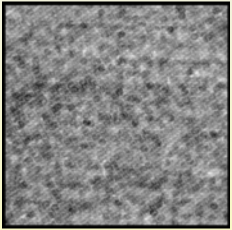
original image



log-likelihood



log-likelihood ratio

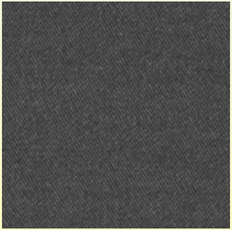


Remark: Log-likelihood values are highly sensitive to the deviations of grey levels. So e.g. hardly visible light pixels in the left image produce dark spots of window size (central image)

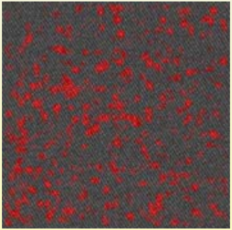
Example 3: Irregularity Evaluation - Texture "Cushion"

Idea: integration of the log-likelihood evaluation into the original image by using red spectral component

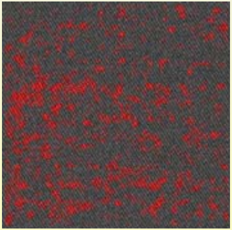
original image



L-strangeness



LR-strangeness



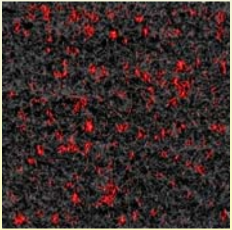
Principle: red spectral component contains the inverse log-likelihood values (central image) and log-likelihood ratio values (right image)

Example 4: Irregularity Evaluation - Texture “Carpet”

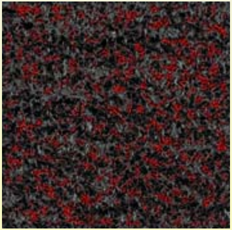
Idea: integration of the log-likelihood evaluation into the original image by using red spectral component

original image

L-strangeness



LR-strangeness



Principle: red spectral component contains the inverse log-likelihood values (central image) and log-likelihood ratio values (right image)

Other examples

Invariance with Respect to Grey-Level Transformation

Invariance Property:

log-likelihood image is invariant with respect to arbitrary linear transform of the grey scale of the original image

the transformed data and transformed mixture parameters

$$y_n = ax_n + b, \quad \tilde{\mu}_{mn} = a\mu_{mn} + b, \quad \tilde{\sigma}_{mn} = a\sigma_{mn}, \quad \mathbf{y} = T(\mathbf{x}), \ \mathbf{x} \in \mathcal{S}$$

can be shown to satisfy the EM iteration equations

$$q(m|\mathbf{y}) = q(m|\mathbf{x}), \quad \mathbf{x} \in \mathcal{S}, \quad \tilde{w}_m = w_m, \quad m \in \mathcal{M}$$

$$F(\mathbf{y}|\tilde{\boldsymbol{\mu}}_m, \tilde{\boldsymbol{\sigma}}_m) = \frac{1}{a^N} F(\mathbf{x}|\boldsymbol{\mu}_m, \boldsymbol{\sigma}_m), \quad \tilde{P}(\mathbf{y}) = \frac{1}{a^N} P(\mathbf{x})$$

and the corresponding log-likelihood values differ only by a constat which is removed by fixing the displayed grey-level interval

$$\log \tilde{P}(\mathbf{y}) = -N \log a + \log P(\mathbf{x}), \quad \mathbf{x} \in \mathcal{S}$$

Computational Details of Texture Evaluation

- log-likelihood image is invariant with respect to linear transforms of the grey scale
- log-likelihood criterion optimally "fits" the estimated mixture to the data set $\mathcal{S}$
- $\Rightarrow$ application of the mixture model to the source data $\mathbf{x} \in \mathcal{S}$ is justified by the estimation procedure
- $\Rightarrow$ log-likelihood value is a suitable measure of typicality of data vectors $\mathbf{x} \in \mathcal{S}$
- unlike other fields (e.g. texture modelling, prediction, pattern recognition) the estimated mixture is applied to the "training" data set $\mathcal{S}$ again
- limited representativeness of the set $\mathcal{S}$ is less relevant since the estimated mixture is not applied to the data not contained in $\mathcal{S}$

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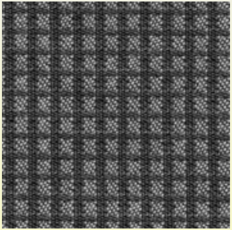
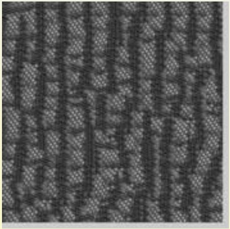
Example 2: Synthesis for Texture "Fabrik"

Synthesis: by random sampling the component means $\boldsymbol{\mu}_{mC}$ according to the conditional weights $W_m(\mathbf{x}_D)$

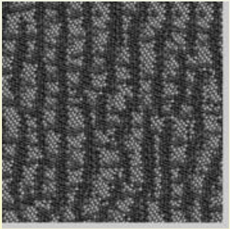
original image



sampling of means



"centroid" sampling



right image: component means replaced by similar pieces of the original texture ("centroids", "micro-tiles") ⇒ stochastic tiling

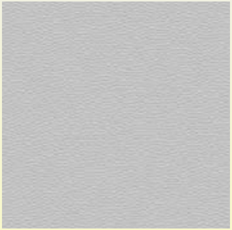
- source texture image: 512x512 pixels $\Rightarrow$ $|\mathcal{S}| \doteq 232000$
- dimension: $N = 30x30 = 900$, number of components: $|\mathcal{M}| = 90$
- number of EM iterations: $t = 20$

Example 3: Synthesis for Texture "Leather"

Synthesis: by random sampling the component means $\boldsymbol{\mu}_{mC}$ according to the conditional weights $W_m(\mathbf{x}_D)$

original image

sampling of means



"centroid" sampling

right image: component means replaced by similar pieces of the original texture ("centroids", "micro-tiles") $\Rightarrow$ stochastic tiling

- source texture image: 512x512 pixels $\Rightarrow$ $|\mathcal{S}| \doteq 242000$
- dimension: $N = 20x20 = 400$, number of components: $|\mathcal{M}| = 50$
- number of EM iterations: $t = 15$

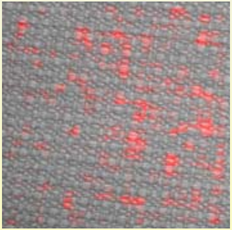
Return

Example 5: Irregularity Evaluation - Texture "Cloth"

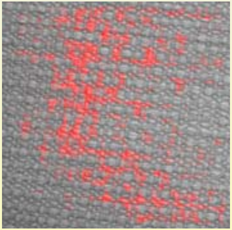
Idea: integration of the log-likelihood evaluation into the original image by using red spectral component

original image

L-strangeness



LR-strangeness

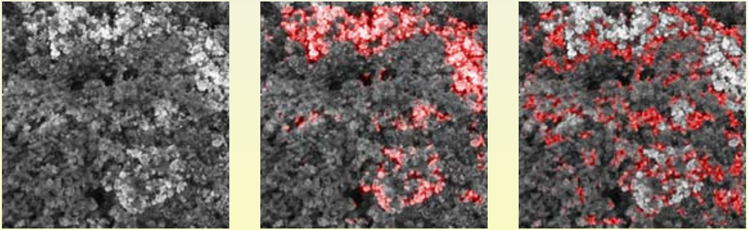


Principle: red spectral component contains the inverse log-likelihood values (central image) and log-likelihood ratio values (right image) Return

Example 6: Irregularity Evaluation - Texture “Flowers”

Idea: integration of the log-likelihood evaluation into the original image by using red spectral component

original image | L-strangeness | LR-strangeness

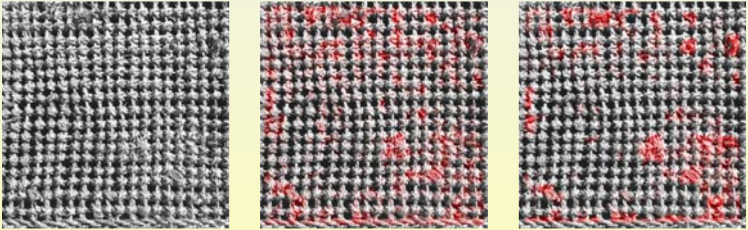


Principle: red spectral component contains the inverse log-likelihood values (central image) and log-likelihood ratio values (right image) Return

Example 7: Irregularity Evaluation - Texture “Ratan”

Idea: integration of the log-likelihood evaluation into the original image by using red spectral component

original image | L-strangeness | LR-strangeness



Principle: red spectral component contains the inverse log-likelihood values (central image) and log-likelihood ratio values (right image) Return